QUANTUM COMPUTING IN PLATO'S CAVE

7/6/14 CEQIP

DANIEL KLAUS BURGARTH

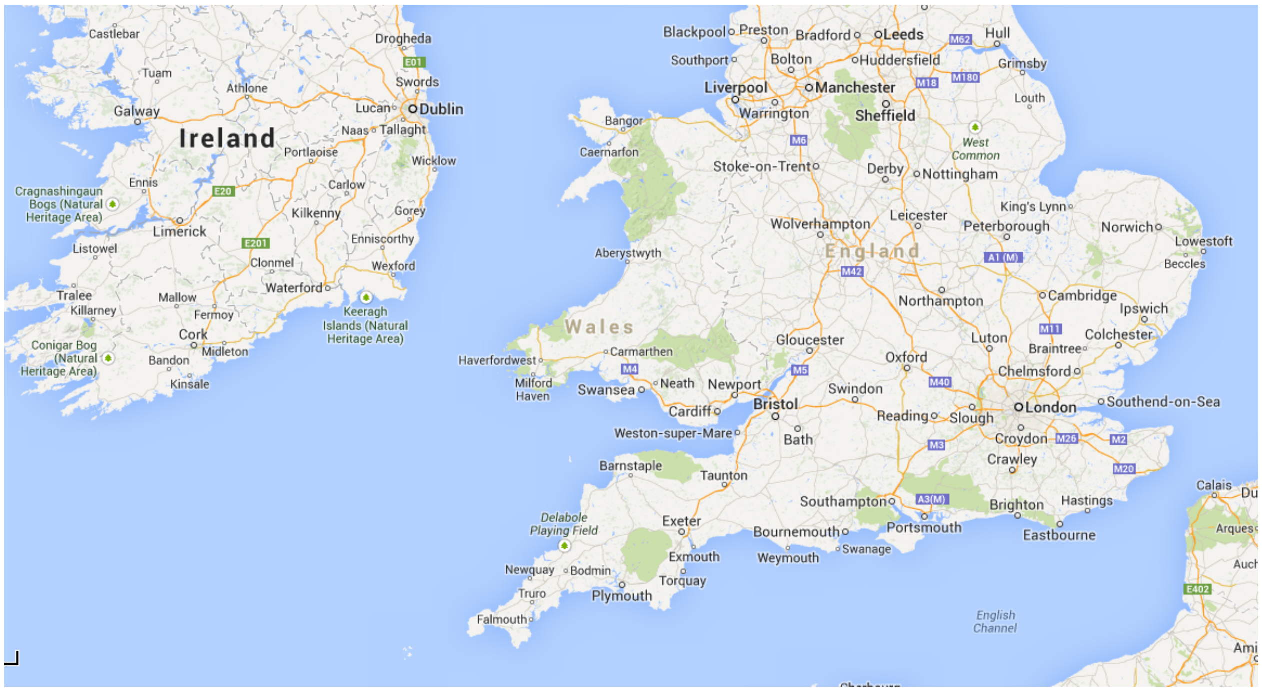
ABERYSTWYTH UNIVERSITY

WITH VITTORIO GIOVANNETTI, PAOLO FACCHI, SAVERIO PASCAZIO, KAZUYA YUASA, HIROMICHI NAKAZATO

1403.5752

ABER WHAT?

ABER WHAT ?



Ireland
Castlebar
Drogheda
Tuam
Athlone
Swords
Galway
Lucan
Dublin
Naas
Tallaght
Portlaoise
Wicklow
Ennis
Carlow
Cragnashingaun Bogs (Natural Heritage Area)
Limerick
Kilkenny
Gorey
Listowel
Enniscorthy
Clonmel
Wexford
Waterford
Tralee
Mallow
Killarney
Fermoy
Keeragh Islands (Natural Heritage Area)
Cork
Conigar Bog (Natural Heritage Area)
Midleton
Bandon
Kinsale
Wales
England
Blackpool
Preston
Bradford
Leeds
Hull
Southport
Bolton
Huddersfield
Grimsby
Liverpool
Manchester
Warrington
Sheffield
Louth
Bangor
Caernarfon
West Common
Stoke-on-Trent
Derby
Nottingham
King's Lynn
Wolverhampton
Leicester
Peterborough
Norwich
Lowestoft
Beccles
Aberystwyth
Northampton
Cambridge
Ipswich
Gloucester
Luton
Braintree
Colchester
Carmarthen
Haverfordwest
Oxford
Chelmsford
Milford Haven
Swansea
Neath
Newport
Swindon
Southend-on-Sea
Cardiff
Bristol
Reading
Slough
London
Weston-super-Mare
Bath
Croydon
Crawley
Barnstaple
Taunton
Calais
Southampton
Brighton
Hastings
Delabole Playing Field
Exeter
Bournemouth
Portsmouth
Eastbourne
Exmouth
Weymouth
Swanage
Newquay
Bodmin
Torquay
Truro
Plymouth
Falmouth
English Channel
E01
E20
E201
E402
M6
M62
M18
M180
M42
M4
M5
M40
M11
M3
M25
M2
M20
A1 (M)
A3(M)

ABER WHAT ?

ABER YSTWYTH

ABER WHAT ?

$\underbrace{\text{ABER}}_{\text{MOUTH}}$ YSTWYTH

ABER WHAT ?

$\underbrace{\text{ABER}}_{\text{MOUTH}}$ $\underbrace{\text{YSTWYTH}}_{\text{A RIVER}}$

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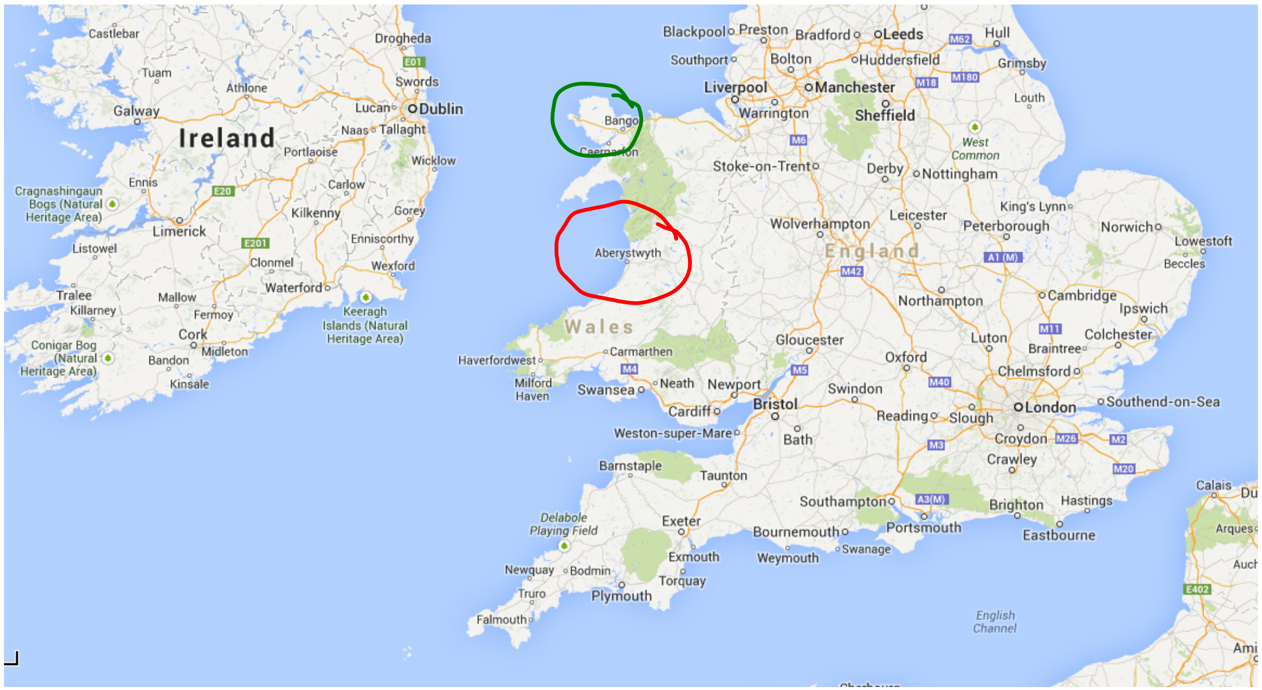
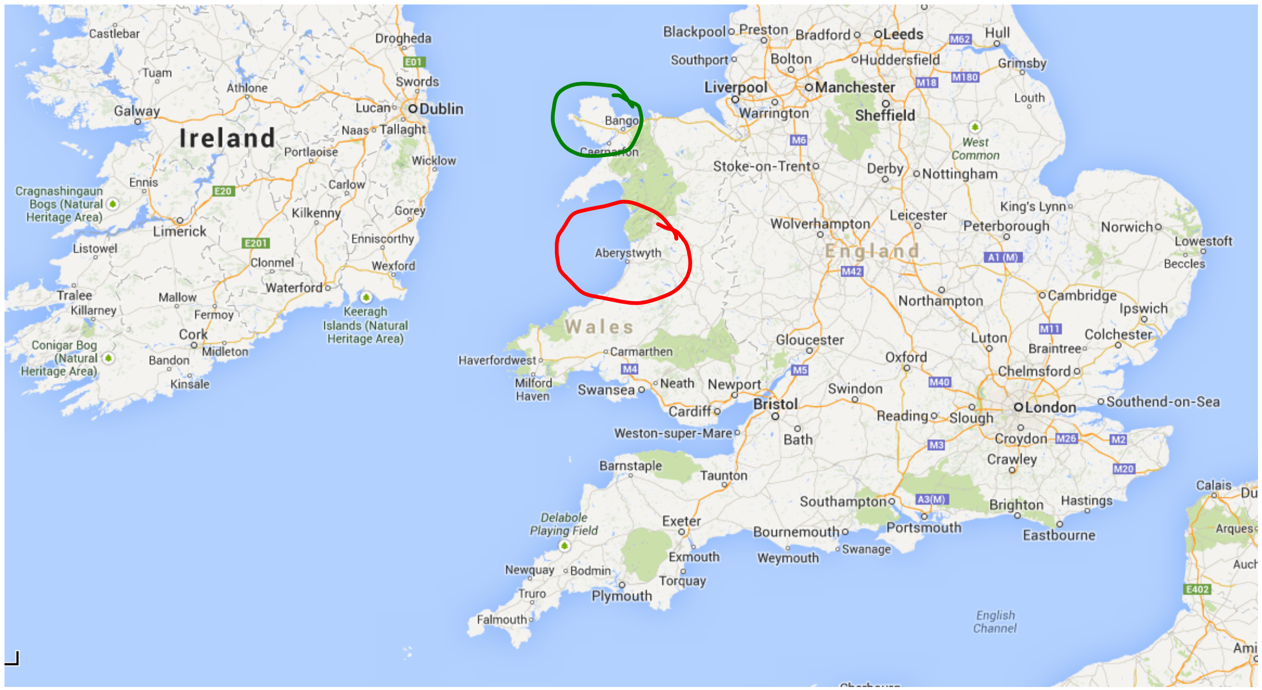


ABER YSTWYTH

MOUTH A RIVER

ABER WHAT ?

Llanfairpwllgwyngyllgogerychwyrndrobwllllantysiliogogogoch



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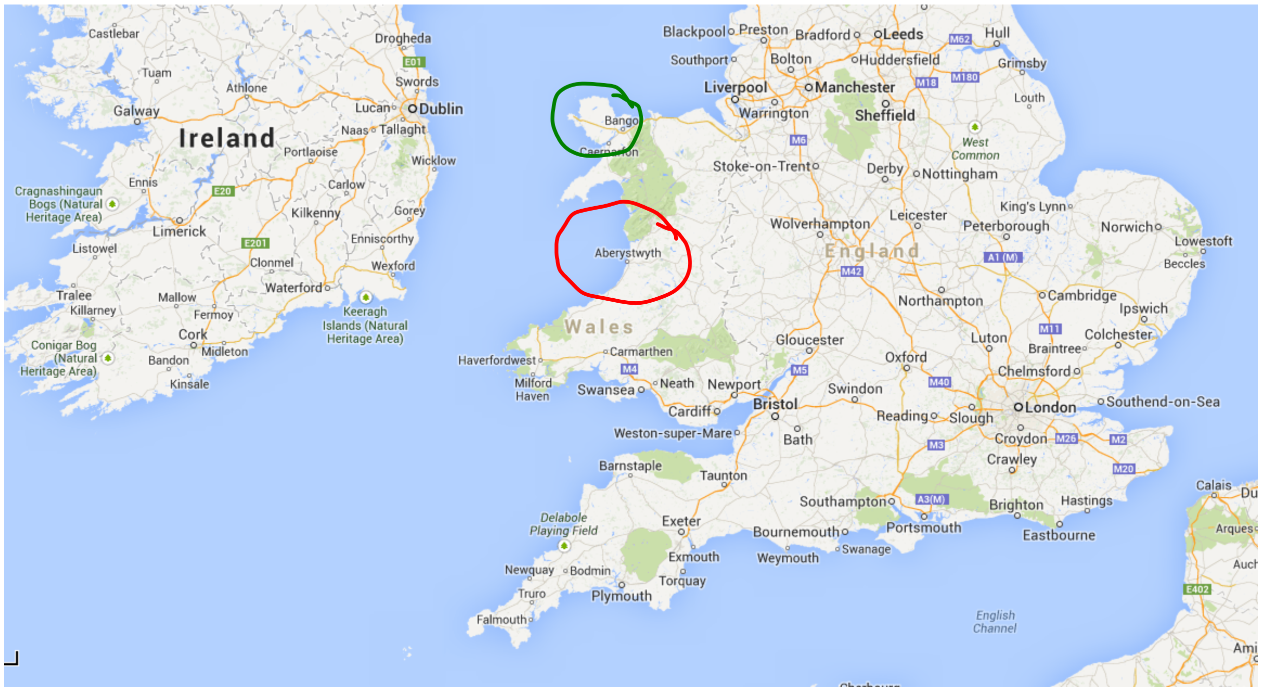
MOUTH A RIVER

ABER WHAT ?

Llanfairpwllgwyngyllgogerychwyrndrobwllllantysiliogogogoch

In Short; Llanfairpwllgwyngyll



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ABER WHAT ?

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$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

ABER WHAT ?

OUTLINE

PLATO'S CAVE

MEASUREMENT IN QM

COMPLEXITY OF DYNAMICS

QUANTUM ZENO

HAMILTONIAN PURIFICATION

ZENO MEETS PLATO

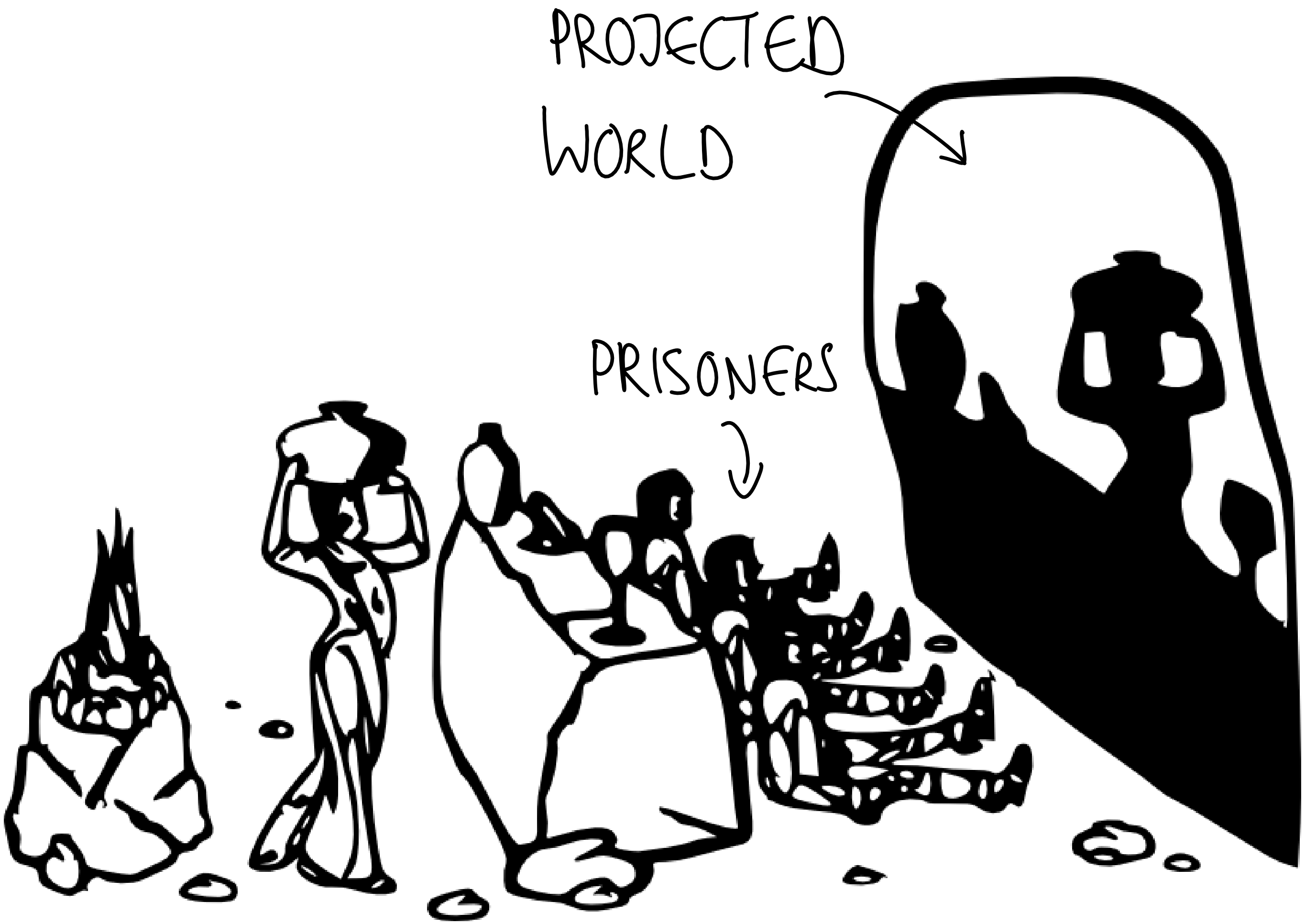
CONCLUSIONS

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE

PLATO'S CAVE



PLATO'S CAVE

PLATO'S CAVE

PROJECTIONS ARE POWERFUL :

PLATO'S CAVE

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PLATO'S CAVE

PROJECTIONS ARE POWERFUL :

IN THIS TALK : QUANTUM VERSION

QUANTUM PROJECTIONS

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

UNCERTAINTY PRINCIPLE

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: BACKACTION

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UNCERTAINTY PRINCIPLE

SUITABLE INITIAL STATE:

QUANTUM COMPUTING

QUANTUM PROJECTIONS

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UNCERTAINTY PRINCIPLE 

SUITABLE INITIAL STATE:

QUANTUM COMPUTING 

QUANTUM PROJECTIONS

MEASUREMENT POSTULATE: <u>BACKACTION</u>

UNCERTAINTY PRINCIPLE 

SUITABLE INITIAL STATE:

QUANTUM COMPUTING 

$\boxed{\text{HOW DO MEASUREMENTS } \underline{\text{GENERALLY}} \text{ CHANGE A SYSTEM?}}$

QUANTIFYING DYNAMICAL COMPLEXITY

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HARD TASK IN GENERAL;

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CONTROLLED HAMILTONIAN DYNAMICS:

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CONTROLLED HAMILTONIAN DYNAMICS:

$$f(t)\, H_1 + g(t)\, H_2$$

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CONTROLS

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CONTROLLED HAMILTONIAN DYNAMICS:

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CONTROLS

$L(iH_1, iH_2) \equiv$ SMALLEST REAL LIE ALGEBRA CONTAINING iH_1, iH_2

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$$\boxed{\text{WHAT HAPPENS IF WE CAN ALSO APPY } P^2 = P}$$

QUANTUM ZENO EFFECT

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$$\left(1 + \frac{x}{n}\right)^n \longrightarrow e^x$$

QUANTUM ZENO EFFECT

$X_n \to X$:

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X_n, X MATRICES

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$$\left(P \exp\left(\frac{iHt}{n}\right) P\right)^n = \left(P + \frac{iPHPt + O(1/n)}{n}\right)^n$$

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FREQUENT MEASUREMENTS MODIFY THE HAMILTONIAN $H \to PHP$

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COMPLEXITY OF ZENO DYNAMICS

ZENO DYNAMICS TAKES PLACE IN SMALLER SPACE PRP. IS IT <u>SIMPLER</u>?

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$P\mathcal{H}P$. IS IT <u>SIMPLER</u>?

DEPENDS ON RANK OF P:

COMPLEXITY OF ZENO DYNAMICS

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DEPENDS ON RANK OF P:

$P = |\phi\rangle\langle\phi|$ $\rightarrow$ $H_Z = PHP$ TRIVIAL ON $P\mathcal{H}P$

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"ZENO'S PARADOX"

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EXAMPLE $P = |\phi\rangle\langle\phi|_1 \otimes \mathbb{1}_2$, RANK 2:

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"ZENO'S PARADOX"

EXAMPLE $\quad P = |\phi\rangle\langle\phi|_1 \otimes \mathbb{1}_2 \,, \quad$ RANK 2:

$|\phi\rangle$ qubit: $\qquad \langle\phi|X|\phi\rangle = \langle\phi|Z|\phi\rangle = 1/\sqrt{2}$

$H_1 = X_1 X_2$

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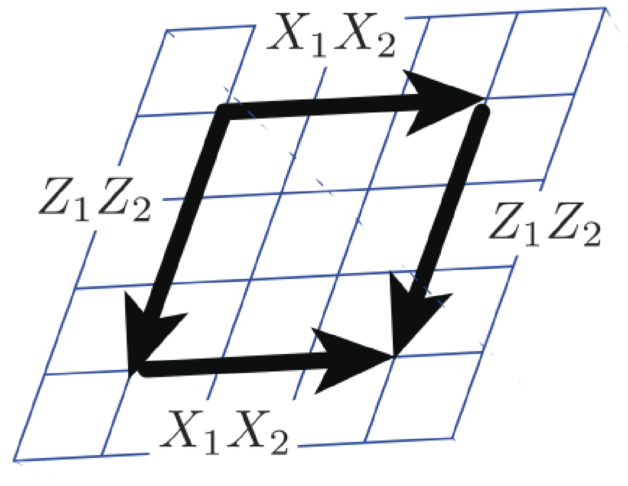

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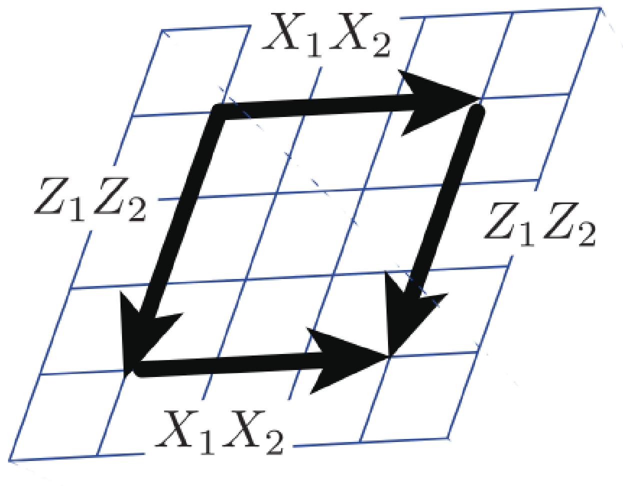

$$P H_1 P \sim P X_2 \equiv H_{z1}$$

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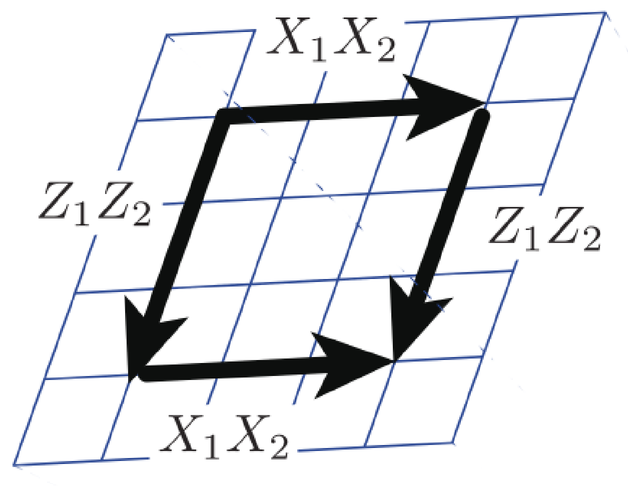

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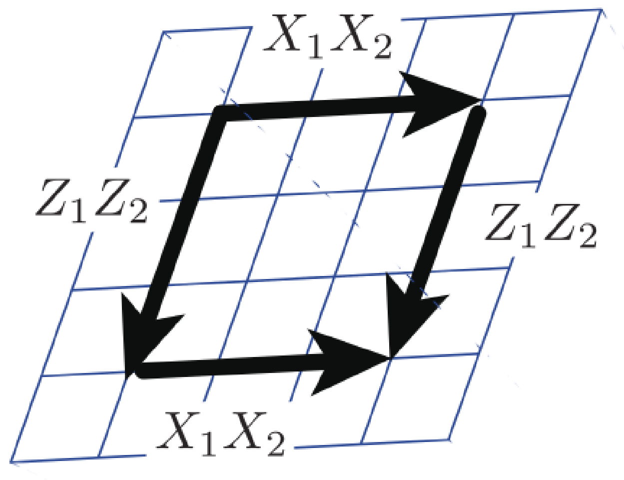

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FREEZING QUBIT 1 IN $|0\rangle$

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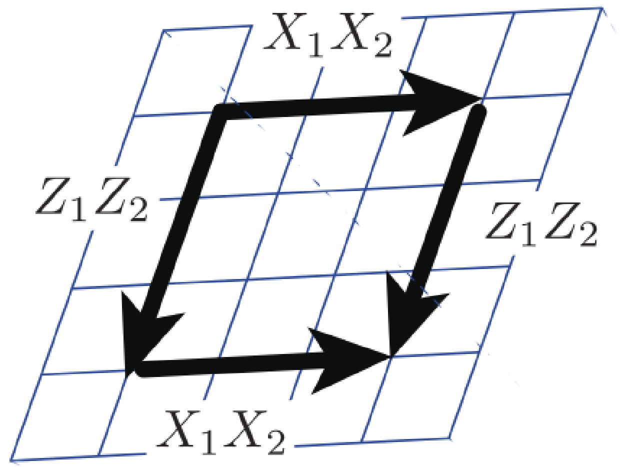

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FREEZING QUBIT 1 IN $|\phi\rangle$

$$[H_{z1}, H_{z2}] \sim P Y_2$$

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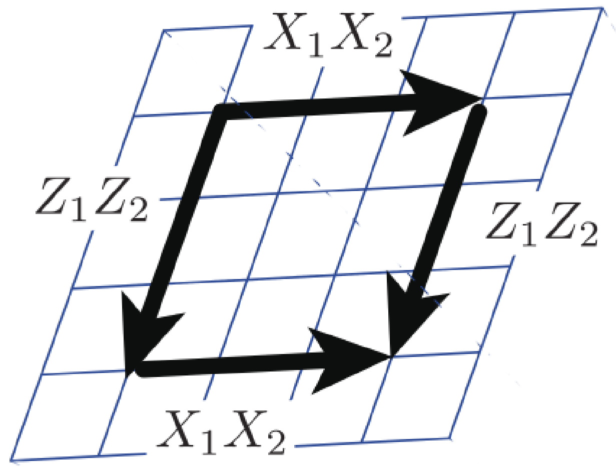

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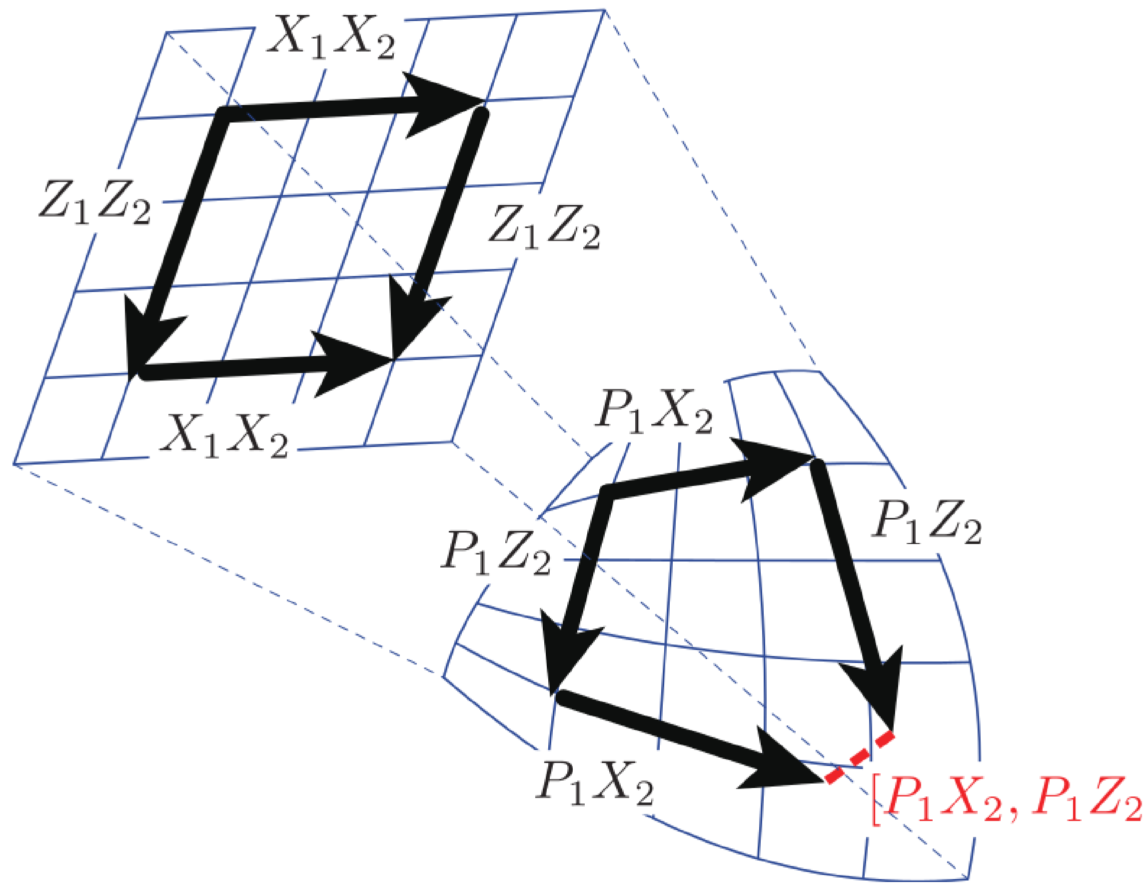
$\rightarrow \dim L_z = 3$, MORE COMPLEX DYNAMICS IN SMALLER SPACE

$$H_1 = X_1 X_2 \qquad \Rightarrow \quad \dim L(iX_1X_2,\ iZ_1Z_2) = 2$$
$$H_2 = Z_1 Z_2$$



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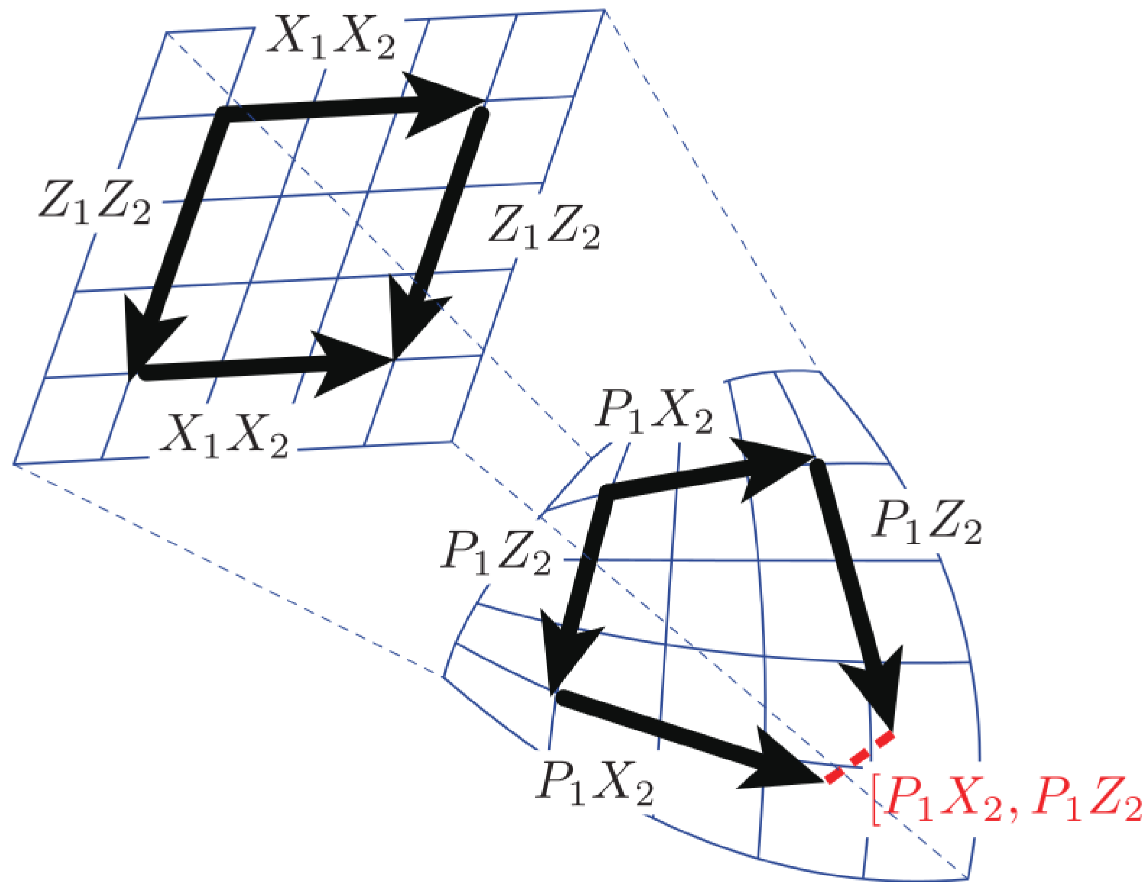

→ $\dim L_z = 3$, MORE COMPLEX DYNAMICS IN SMALLER SPACE

→ PROJECTION INTRODUCES CURVATURE

HOW GENERAL IS THIS

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<u>PURIFICATION LEMMA:</u> h_1, h_2 ON $d \times d$ GENERIC

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$$H_1 \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes h_1 + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \otimes h_2$$

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$$\Rightarrow [H_1, H_2] =$$

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$$\Rightarrow [H_1, H_2] = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \otimes [h_1, h_2] + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}^2 \otimes [h_2, h_1]$$

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$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1}$$

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$$P \equiv \begin{pmatrix} 1 & \\ & \end{pmatrix} \otimes \mathbb{1} \quad \Rightarrow \quad P H_1 P = P \otimes h_1$$

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"HAMILTONIAN PURIFICATION"

NON-COMMUTATIVE BECOMES SIMPLE ON LARGER SPACE

FREQUENT MEASUREMENTS BRING US BACK

$\boxed{\text{KURANISHI ('52)}: \exists\ h_1, h_2 : \ L = u(d) \text{ MAXIMAL}}$

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INTO A QUANTUM COMPUTER

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$\Rightarrow$ QUANTUM PLATO CAVE

HOW MIGHT THESE LOOK LIKE

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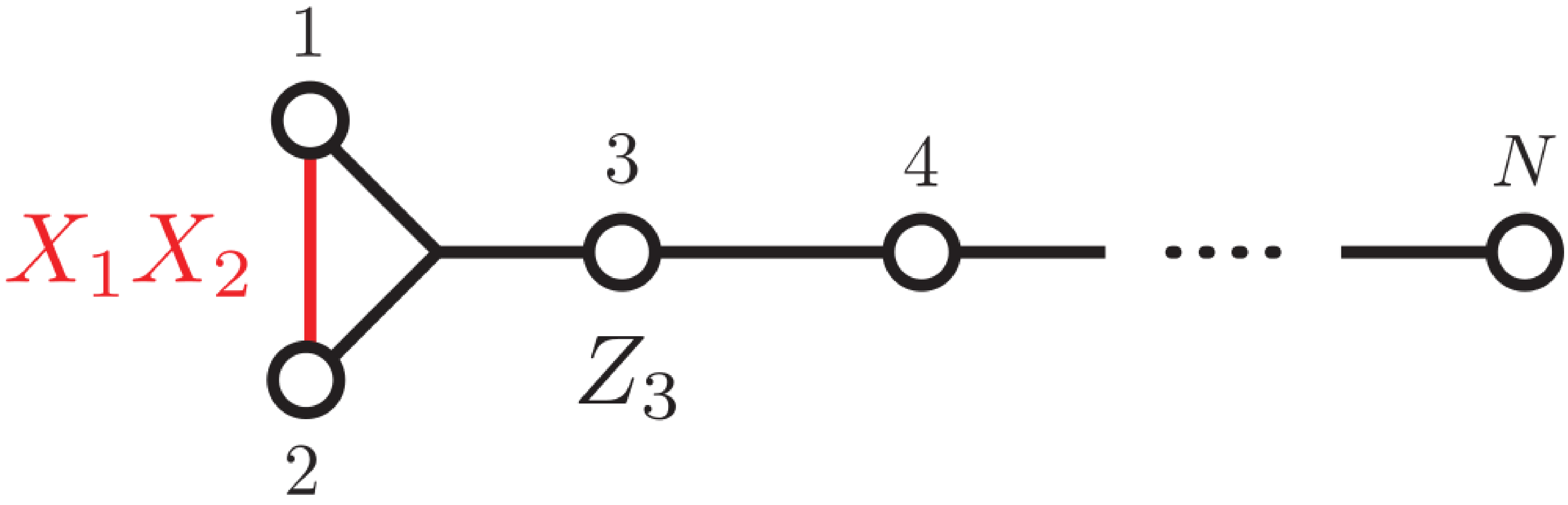

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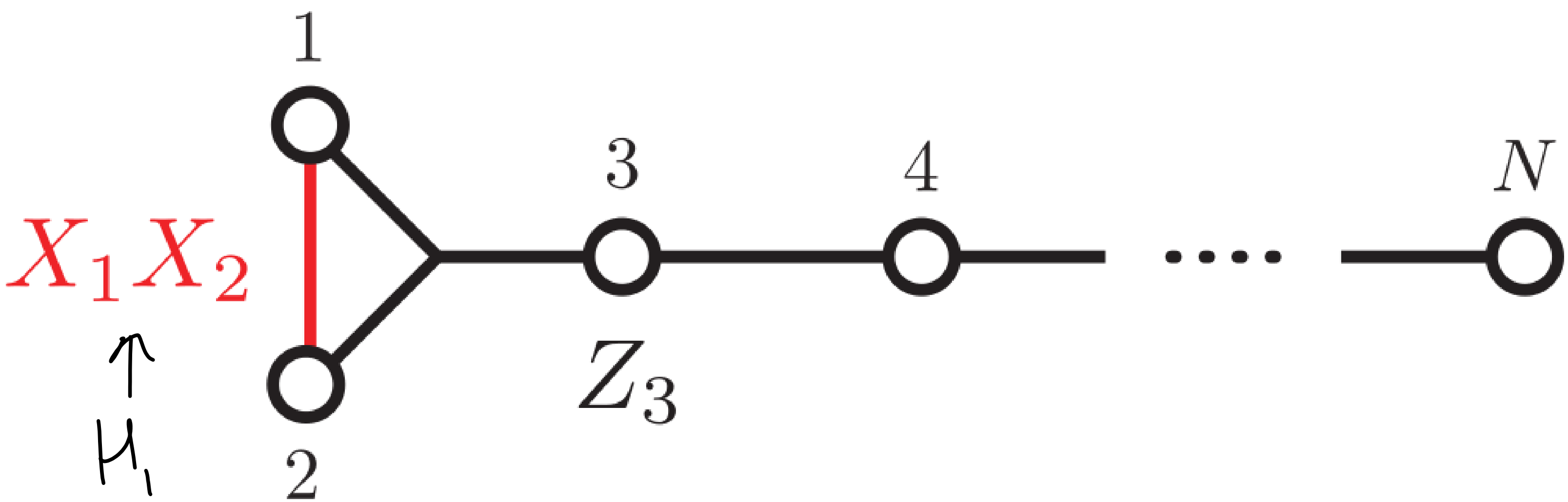

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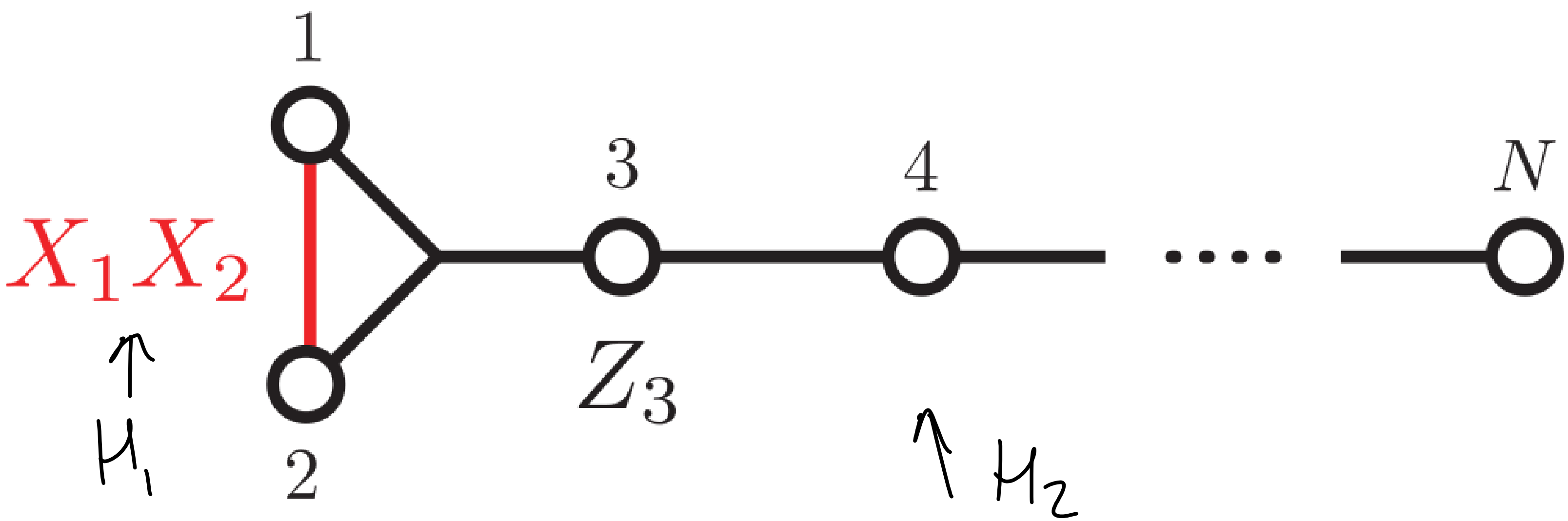

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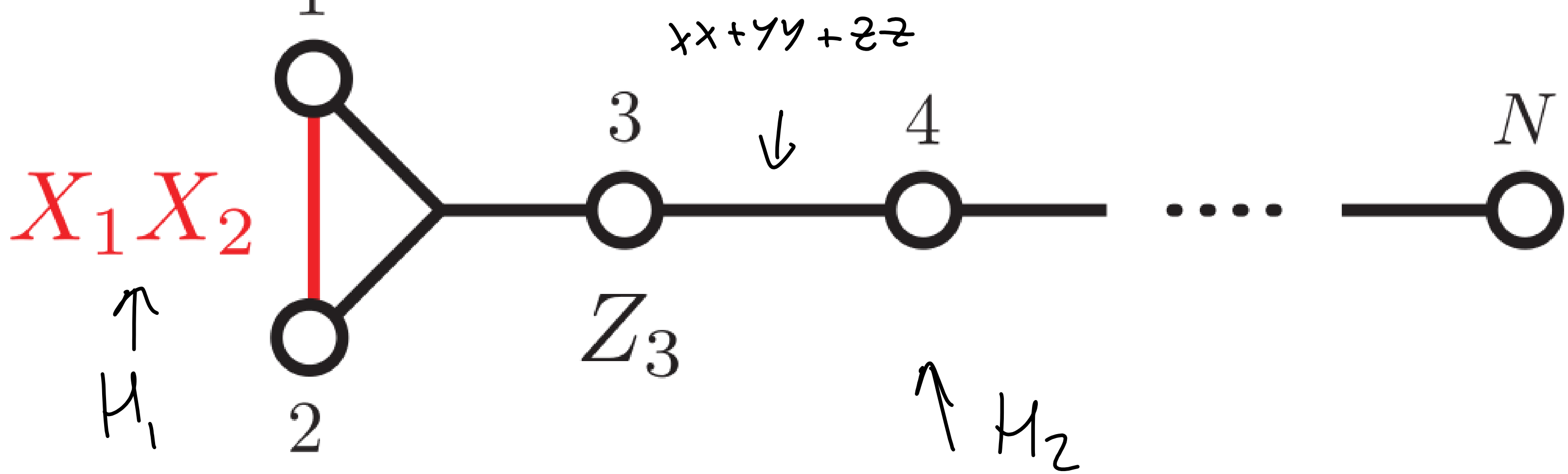

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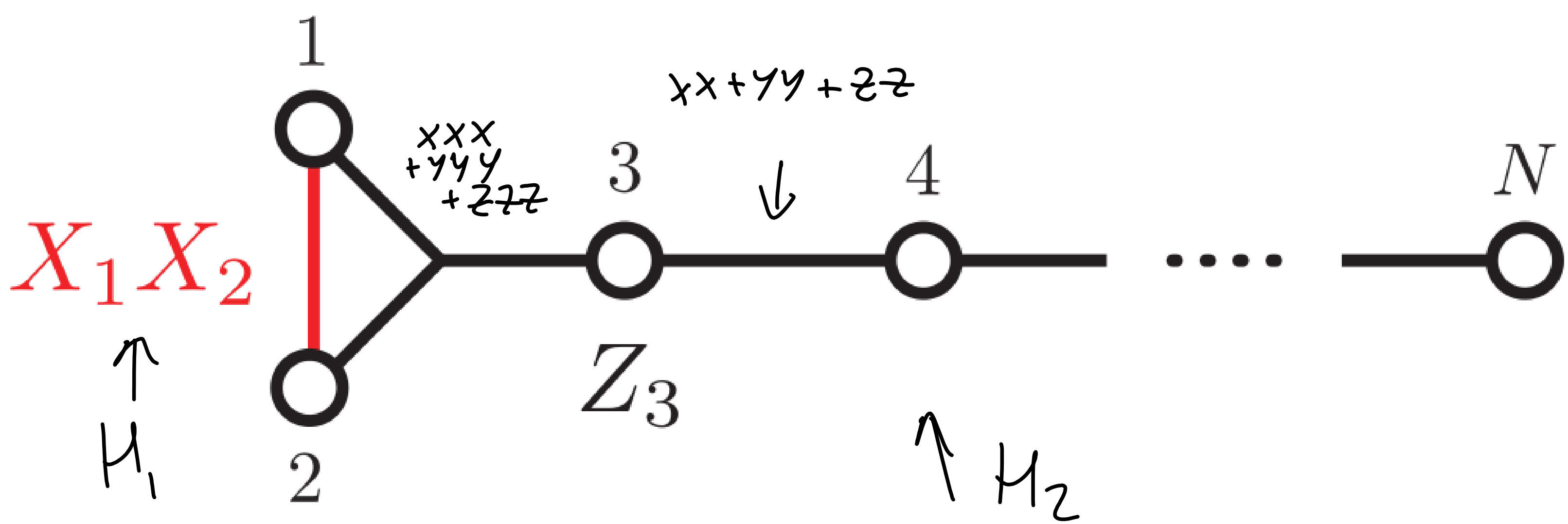
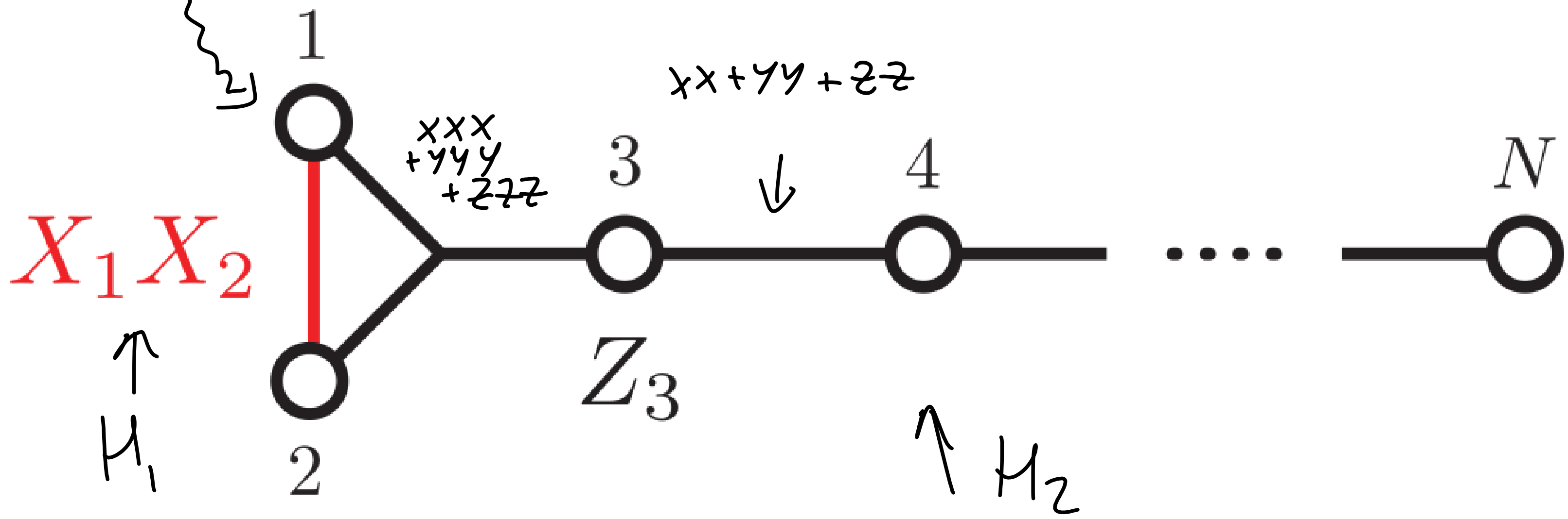


HOW MIGHT THESE LOOK LIKE

MEASURE 1 QUBIT IN $\langle \phi | \{X, Y, Z\} | \phi \rangle = \frac{1}{\sqrt{3}}$



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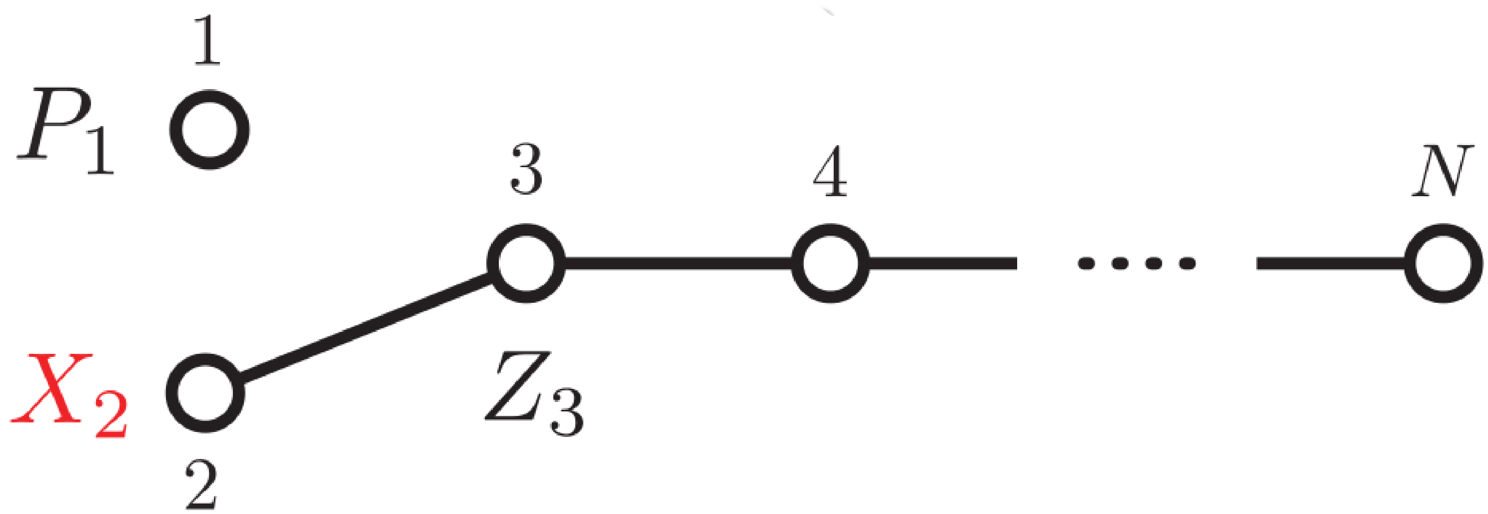

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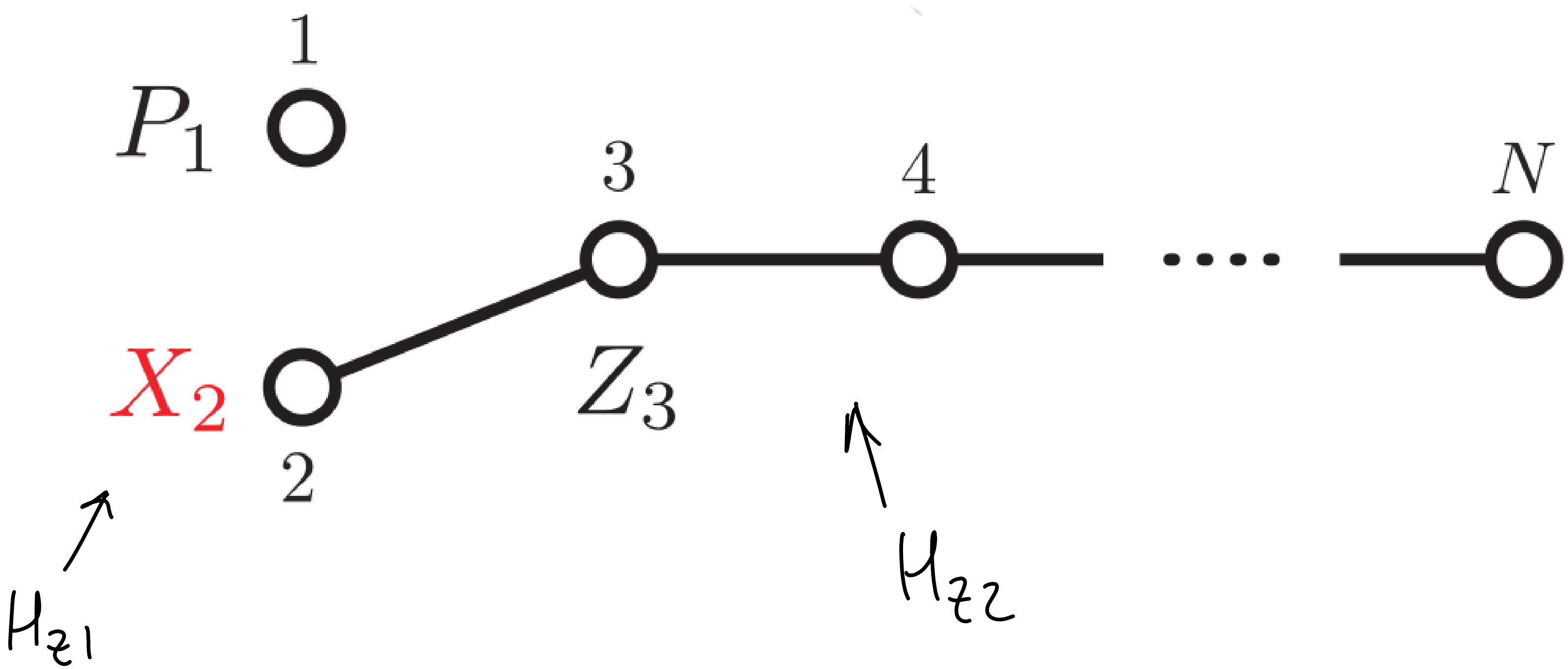

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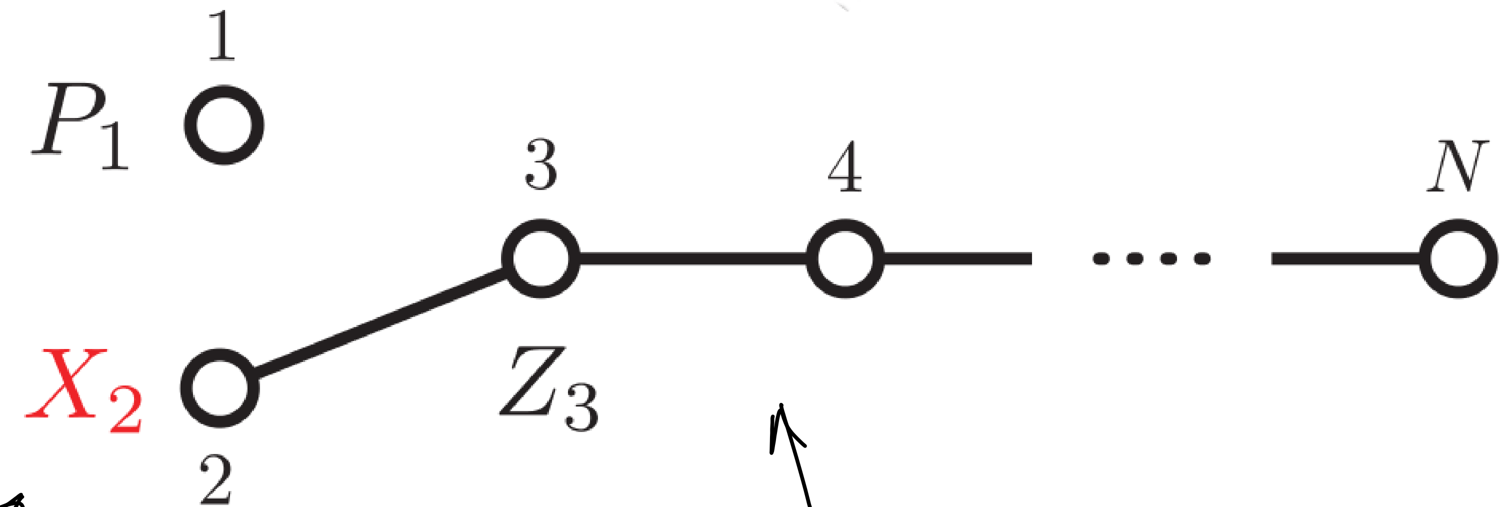

H_{z1}

H_{z2}

... THESE GENERATE $su(2^{N-1})$ ON $2, \ldots, N$

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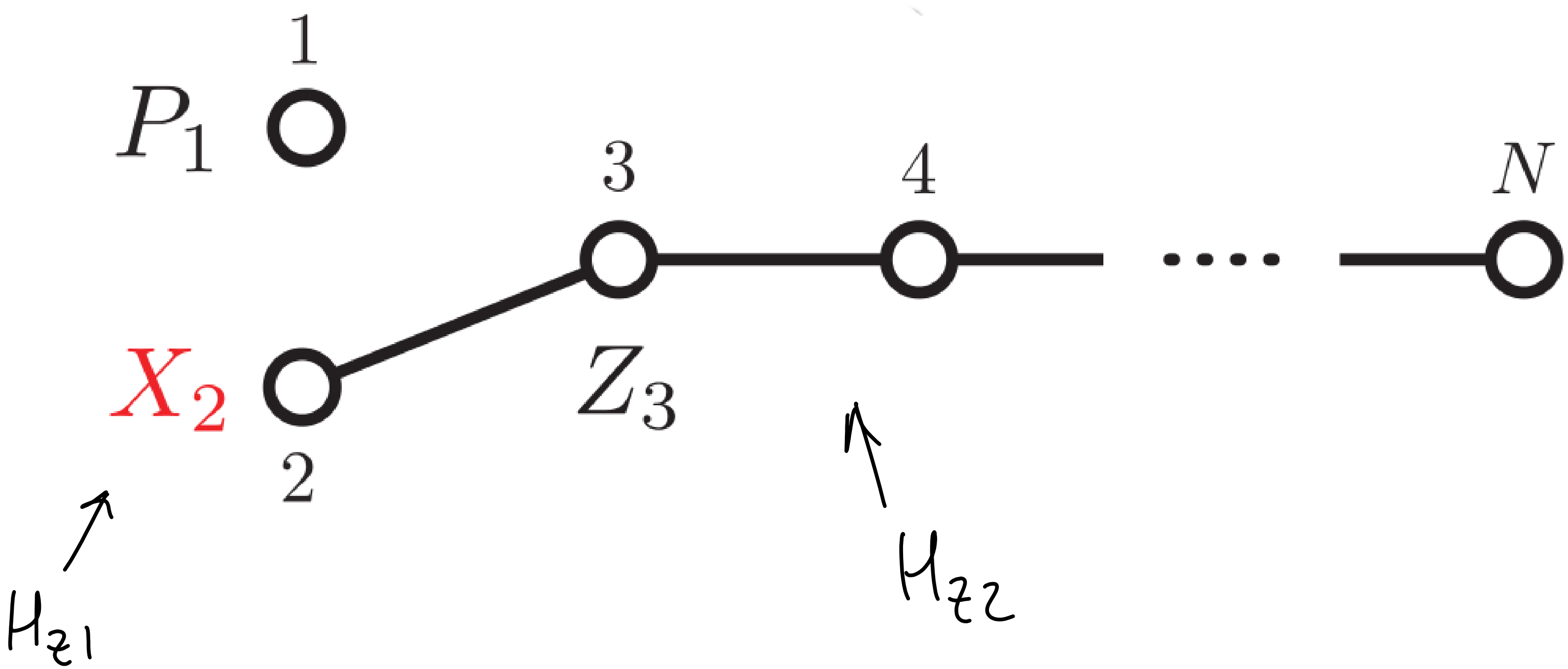

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(ALSO HAVE 2-BODY ONLY EXAMPLES)

FINITE TIME BETWEEN PROJECTIONS

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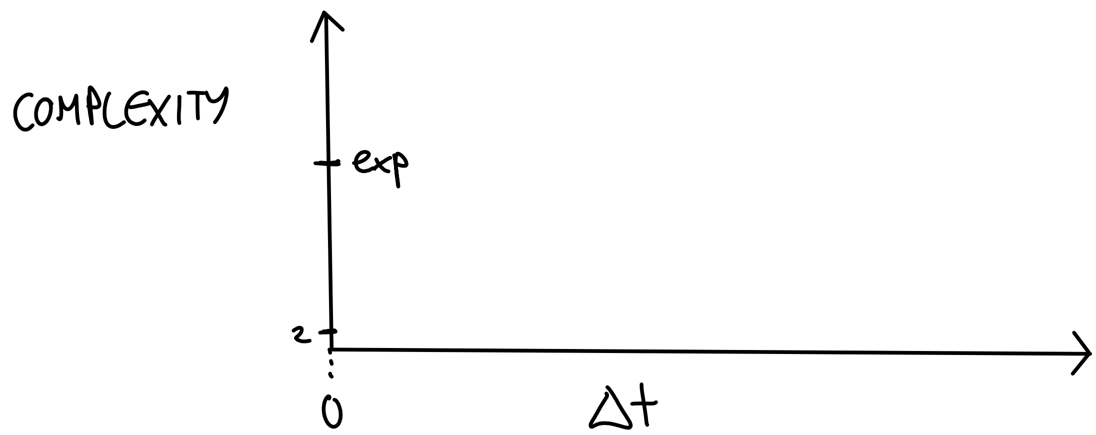

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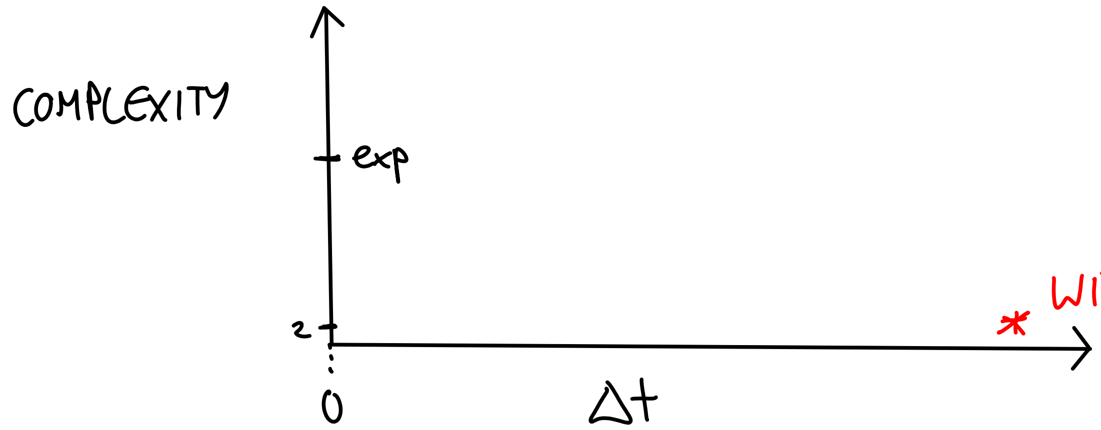
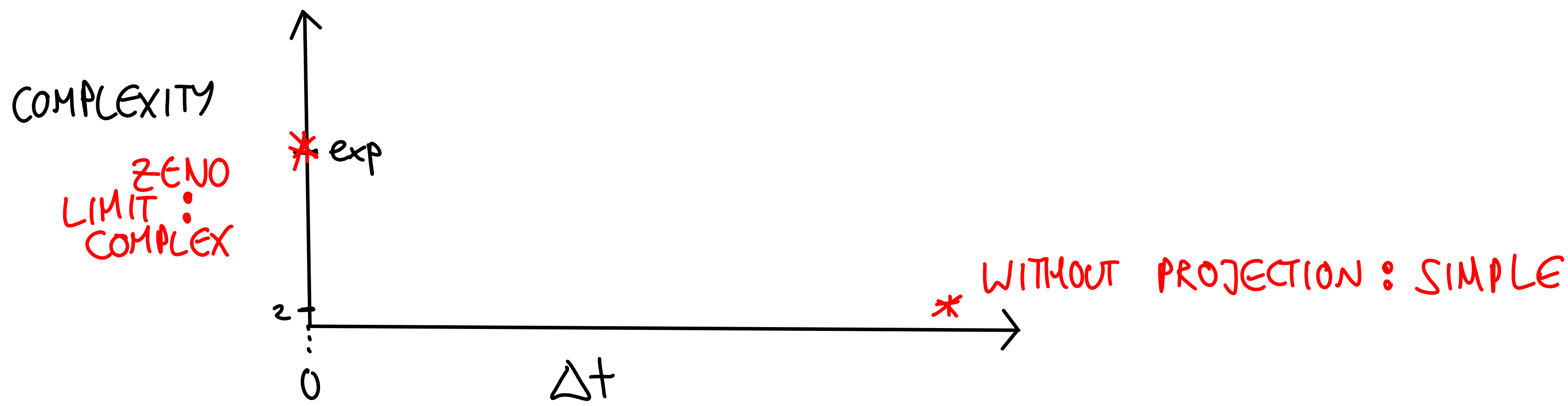


* WITHOUT PROJECTION : SIMPLE

FINITE TIME BETWEEN PROJECTIONS



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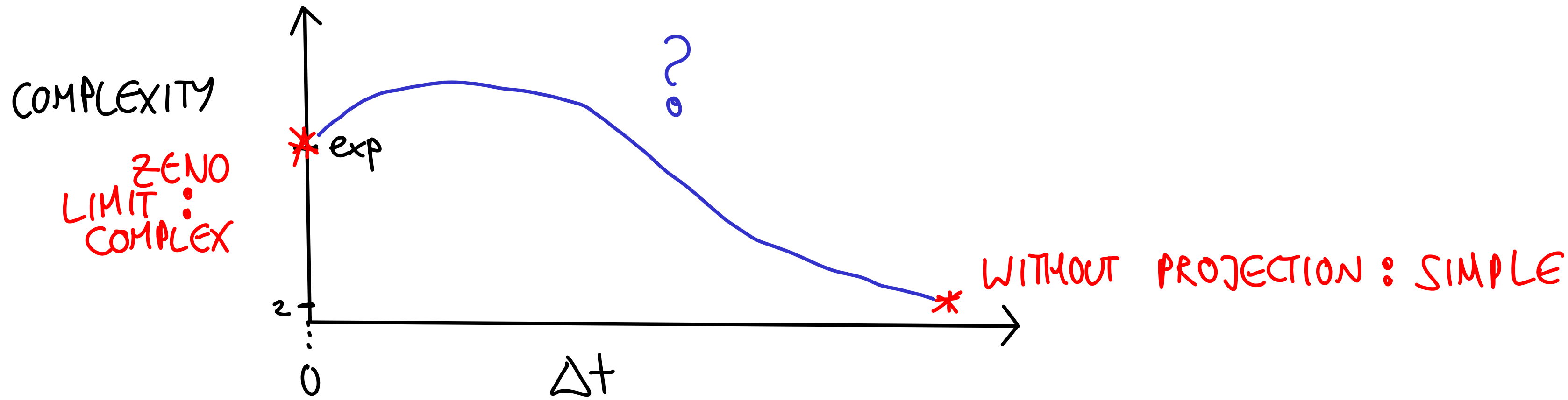

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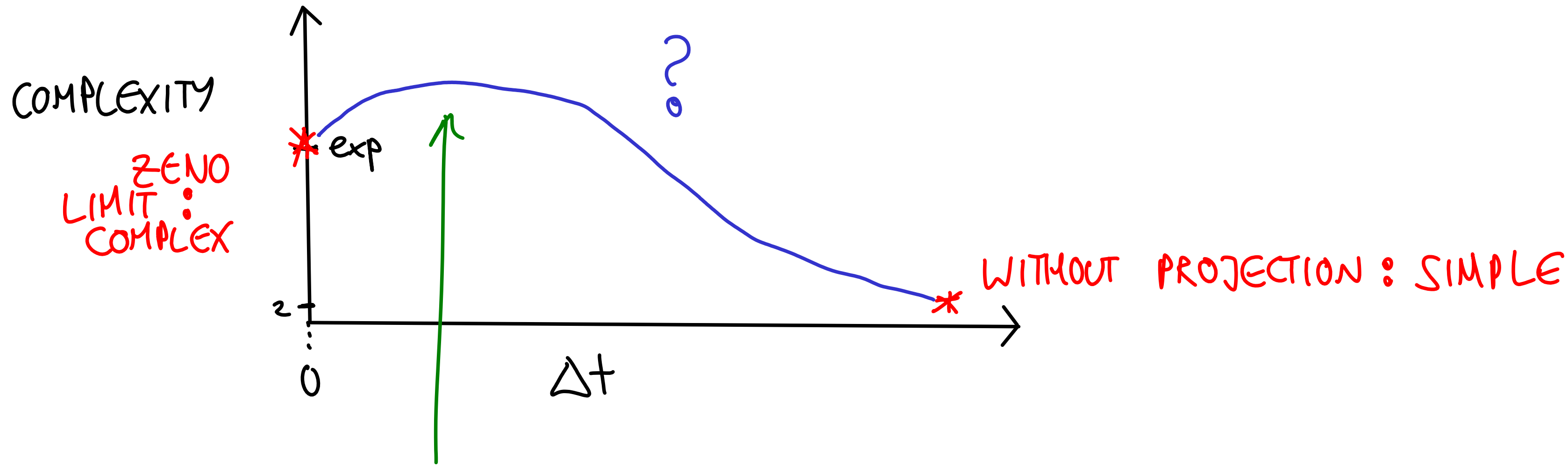

AWAY FROM 0 : ALREADY NON-UNITARY
BUT STILL COMPLEX ??

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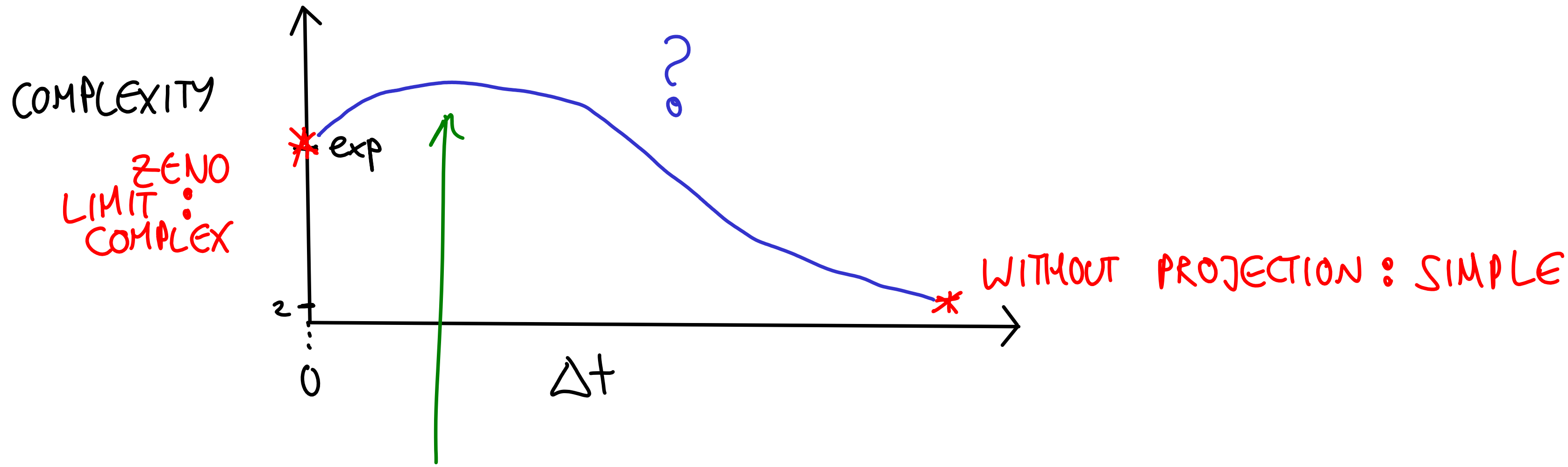

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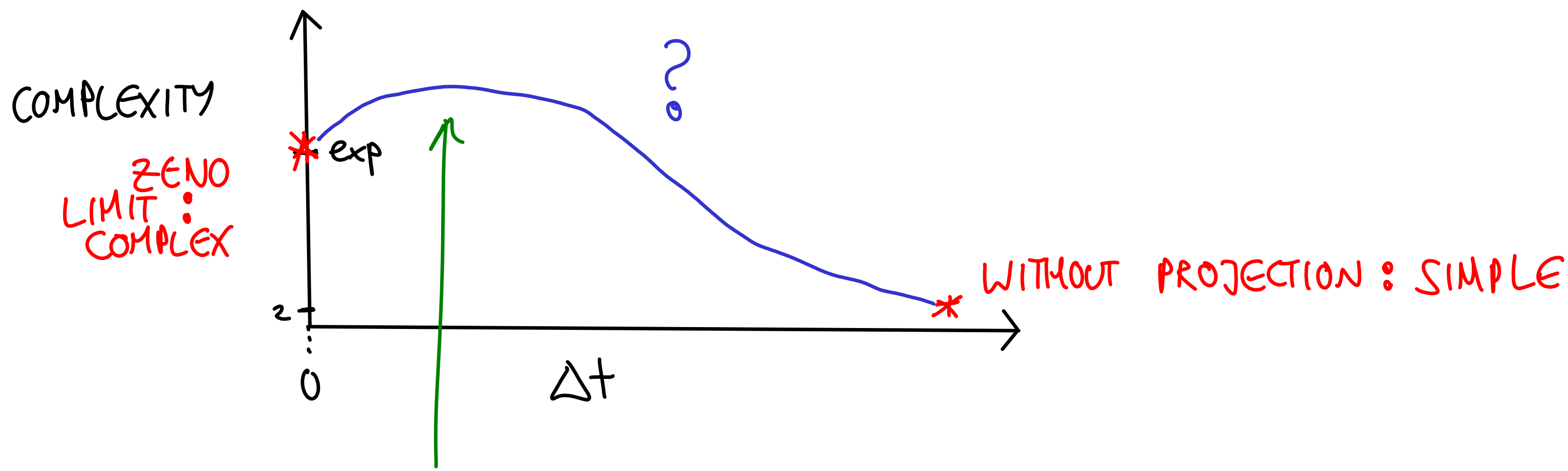

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OTHER IMPLEMENTATIONS

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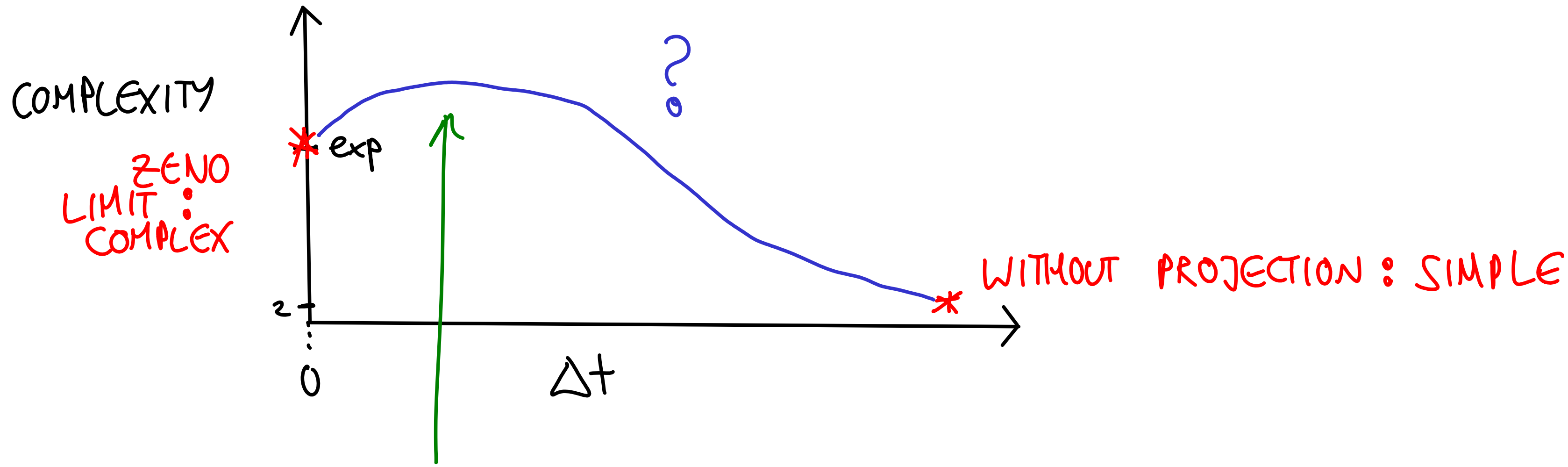

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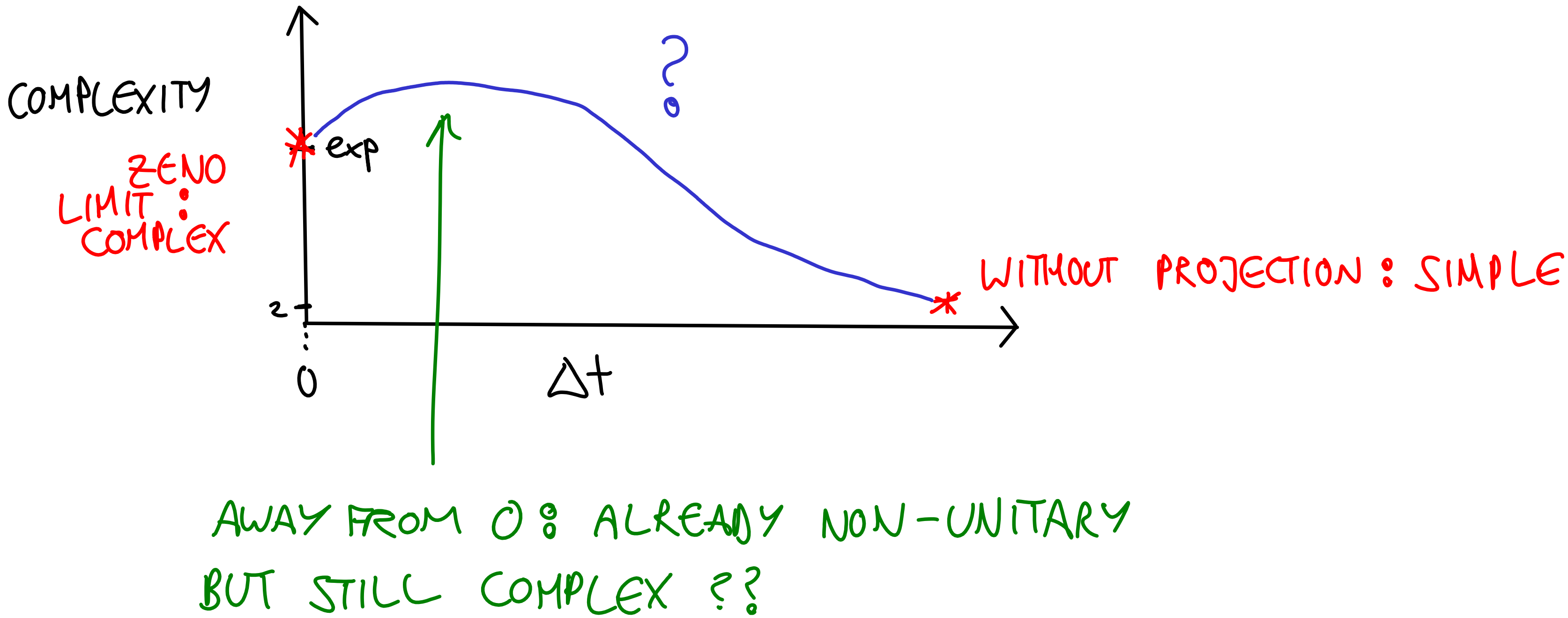

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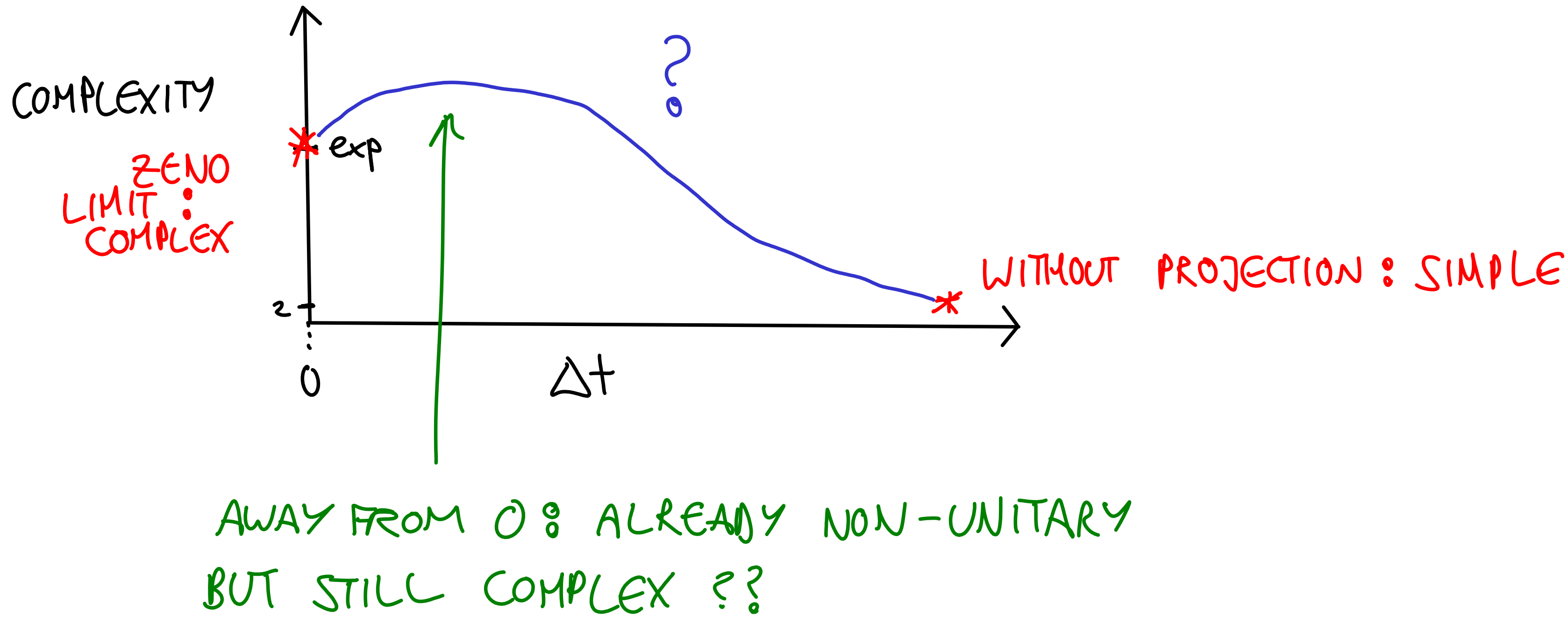

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→

GEOMETRIC CONSTRAINTS IN QM GENERALLY RAISE COMPLEXITY EXPONENTIALLY

CONCLUSIONS

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CONFIRMS POWER OF MEASUREMENT

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BUT WHAT IS A MEASUREMENT ???