Dissociation and annihilation of multipartite entanglement structures

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Based on

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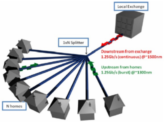
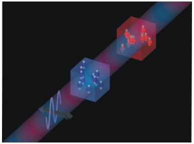
Motivation

Need of preserving multiparticle entanglement:

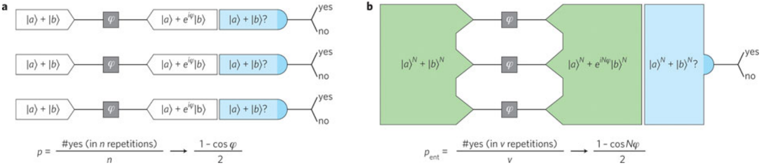
- quantum networking applications (secret sharing & voting, open-destination teleportation, etc.)



- quantum memory devices



- quantum interferometry



Previous effort...

... is the question of measure:

- Negativity $\frac{1}{2}\left(\|\varrho^{\Gamma}\|_1 - 1\right)$
  - Simon *et al*. (2002), Dür *et al*. (2004), Bandyopadhyay *et al*. (2005), Hein *et al*. (2005), Man *et al*. (2008), Aolita *et al*. (2008–2010)
  - simplicity of computation
  - arbitrary number of qubits (except for some graph states and randomly sampled states)
  - sensitivity to the entanglement with respect to a particular bipartition only (remember, e.g., bound-entangled PPT states and biseparable but non-triseparable states)
  - little information about the entanglement structure

Previous effort...

... is the question of measure:

- geometric measure of entanglement
- entropic measure
- ‘average’ SL-invariant entanglement measure

} absence of full separability*

- projector-like witness
- specific generalization of the concurrence
- collective spin-based entanglement witness

} presence of genuine entanglement*

What is the problem?

The problem is to track up the transformations of entanglement structure during the dissipative processes.

Structure:

- the number of separate components
- the number of particles within each of components
- + allowance for convex mixtures

Structuring entanglement: Partitioning

N-body system $ABC\dots$

k subsystems $\qquad S(N,k) = \frac{1}{k!}\sum_{m=0}^{k}(-1)^m\binom{k}{m}(k-m)^N$

3-body system:

$$\mathcal{P}^1(ABC) = \{ABC\},$$
$$\mathcal{P}^2(ABC) = \{A|BC, B|AC, C|AB\},$$
$$\mathcal{P}^3(ABC) = \{A|B|C\}$$

4-body system:

$$\mathcal{P}^1(ABCD) = \{ABCD\},$$
$$\mathcal{P}^2(ABCD) = \{A|BCD,\ B|ACD,\ C|ABD,\ D|ABC,\ AB|CD,\ AC|BD,\ AD|BC\},$$
$$\mathcal{P}^3(ABCD) = \{A|B|CD,\ A|C|BD,\ A|D|BC,\ B|C|AD,\ B|D|AC,\ C|D|AB\},$$
$$\mathcal{P}^4(ABCD) = \{A|B|C|D\}$$

$\mathcal{P}^k_j$ is the j-th partition of the set $\mathcal{P}^k$, for example
$\mathcal{P}^3_5(ABCD) = B|D|AC$

Structuring entanglement: k-separability

Separable states: $\sigma_j^k = \sum_i \mu_i \varrho_i^{[\mathcal{P}_j^k(ABC...)]_1} \otimes \cdots \otimes \varrho_i^{[\mathcal{P}_j^k(ABC...)]_k}$

k-separable state: $\varrho = \sum_{j=1}^{S(N,k)} p_j^k \sigma_j^k$
Notation: $\varrho_{k\text{-sep}}$

Sets of k-separable states:

$$\mathcal{S}_{N\text{-sep}} \subset \ldots \subset \mathcal{S}_{2\text{-sep}} \subset \mathcal{S}_{1\text{-sep}}$$

k-separability measure:

$$K_{\mathrm{sep}}[\varrho] := \max_{\varrho = \varrho_{k\text{-sep}}} k$$

Structure visualization

Structure visualization

Structure visualization

We want more!

Example: $\varrho^{AB} \otimes \varrho^{CDE}$
How can we quantify *the number of entangled particles*?

Resource-intensiveness (entanglement depth, producibility):

$$R_{\text{ent}}[\varrho] := \min_{\varrho = \sum_{k=1}^{N} \sum_{j=1}^{S(N,k)} p_j^k \varrho_j^k} \max_{m=1,\ldots,k} \left\{ \#\text{ bodies within } [\mathcal{P}_j^k]_m \right\}$$

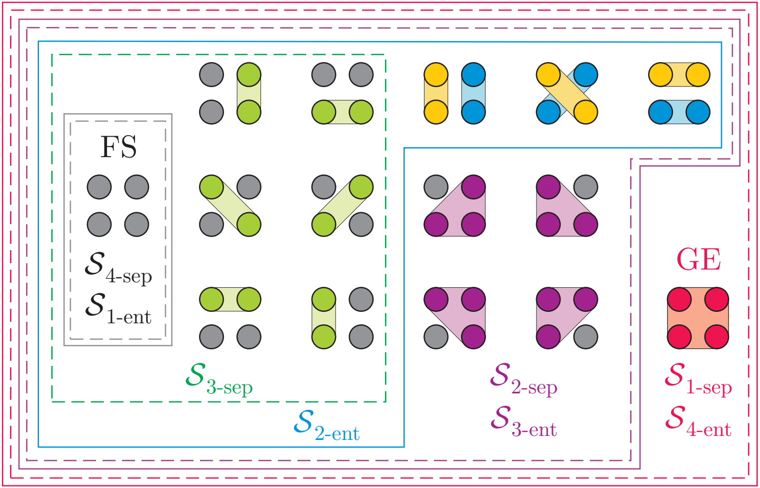
$\mathcal{S}_{r\text{-ent}} = \{\varrho : R_{\text{ent}}[\varrho] \leq r\}$ is the convex set of r-entangled states

$$\mathcal{S}_{1\text{-ent}} \subset \mathcal{S}_{2\text{-ent}} \subset \ldots \subset \mathcal{S}_{N\text{-ent}}$$

Note that $\mathcal{S}_{(N-1)\text{-ent}} = \mathcal{S}_{2\text{-sep}}$

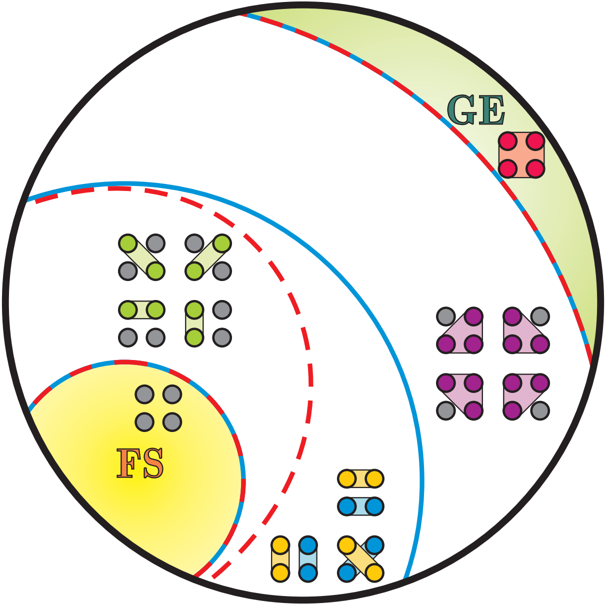
Another family of nested sets

Altogether

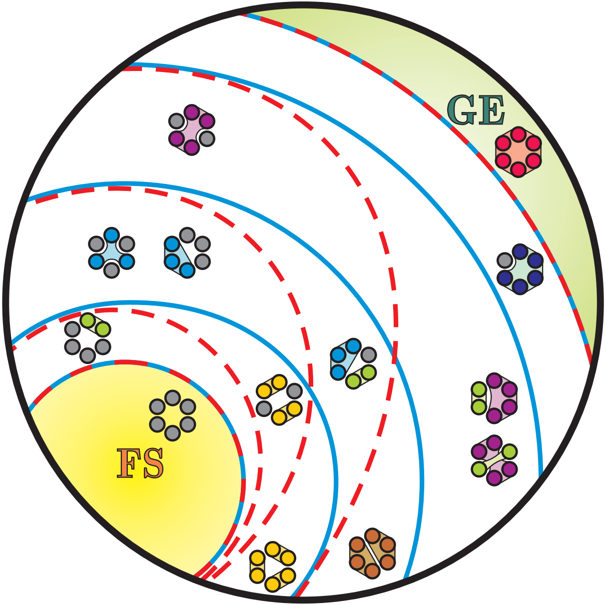


FS
$\mathcal{S}_{4\text{-sep}}$
$\mathcal{S}_{1\text{-ent}}$
$\mathcal{S}_{3\text{-sep}}$
$\mathcal{S}_{2\text{-ent}}$
$\mathcal{S}_{2\text{-sep}}$
$\mathcal{S}_{3\text{-ent}}$
GE
$\mathcal{S}_{1\text{-sep}}$
$\mathcal{S}_{4\text{-ent}}$

Overview of 4-body system entanglement



Overview of 6-body system entanglement



Quantum dynamics: basics

$$\varrho_{\text{out}} = \Phi[\varrho_{\text{in}}]$$

Physical meaning = the Stinespring dilation:

$$\Phi[\varrho_{\text{in}}] \equiv \mathrm{tr}_{\mathrm{env}}[U(\varrho_{\mathrm{in}} \otimes \xi_{\mathrm{env}})U^\dagger]$$

Choi–Jamiołkowski isomorphism:

$$\Omega_\Phi^{SS'} := (\Phi^S \otimes \mathrm{Id}^{S'})[|\Psi_+^{SS'}\rangle\langle\Psi_+^{SS'}|]$$
$$\Phi[X] = d^S \, \mathrm{tr}_{S'} \left[\Omega_\Phi^{SS'} (I_{\mathrm{out}}^S \otimes X^{\mathrm{T}}) \right]$$

$d = \dim \mathcal{H}$
$|\Psi_+^{SS'}\rangle = (d^S)^{-1/2} \sum_{i=1}^{d^S} |i \otimes i'\rangle$

Φ^S is CP if and only if $\Omega_\Phi^{SS'} \geq 0$

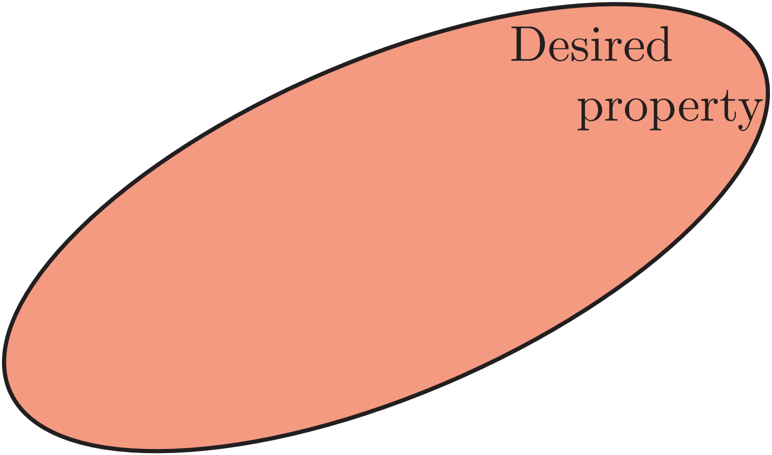
Dissociation

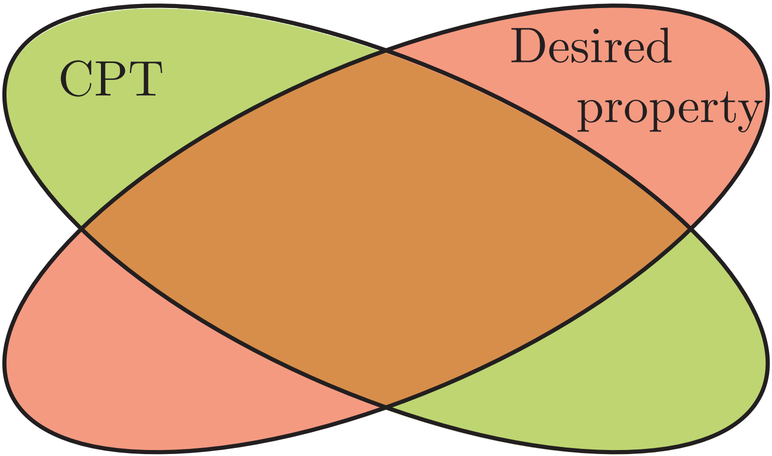
Definition: If ϱ_{out} is separable with respect to the partition $\mathcal{P}_j^k(ABC\ldots)$ (i.e. $\varrho_{\text{out}} = \sigma_j^k$), then we say that the channel Φ *dissociates* entanglement compound of given ϱ_{in} into smaller compounds of $[\mathcal{P}_j^k]_1, \ldots, [\mathcal{P}_j^k]_k$ and denote $\Phi \in \mathcal{D}_j^k(\varrho_{\text{in}})$

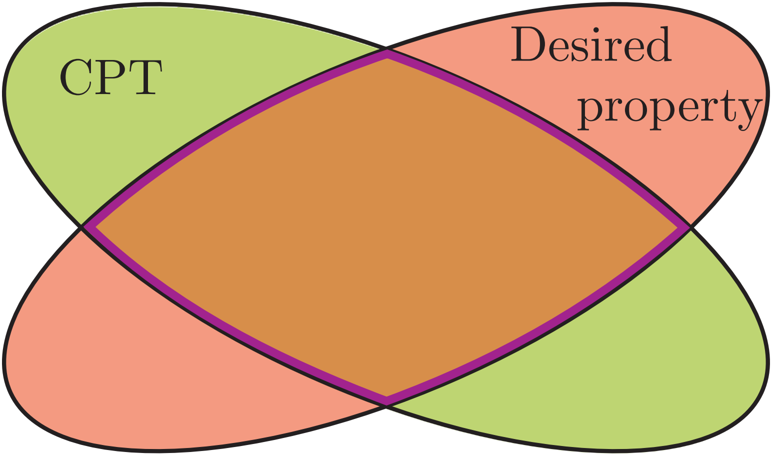
State dependent:
$k\mathsf{Sep}(\varrho_{\text{in}})$ is a set of channels Φ such that $K_{\text{sep}}\left[\Phi[\varrho_{\text{in}}]\right] \geq k$.
$r\mathsf{Ent}(\varrho_{\text{in}})$ is a set of channels Φ such that $R_{\text{ent}}\left[\Phi[\varrho_{\text{in}}]\right] \leq r$.

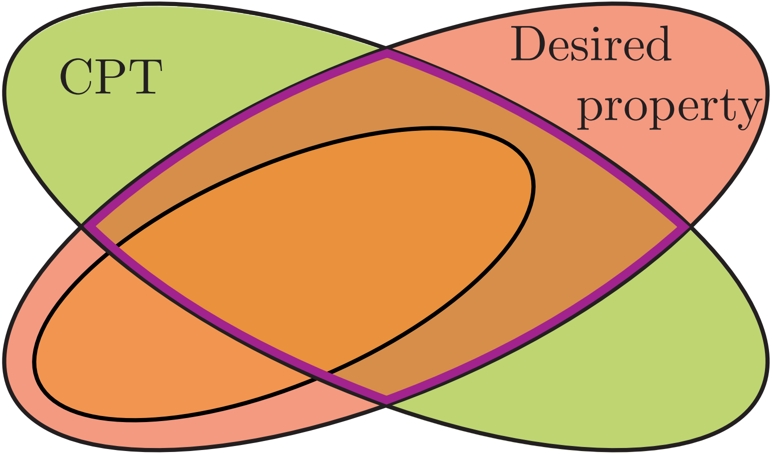
State independent:
$k\mathsf{Sep} := \bigcap_{\varrho_{\text{in}} \in \mathcal{S}(\mathcal{H}^S)} k\mathsf{Sep}(\varrho_{\text{in}})$ and $r\mathsf{Ent} := \bigcap_{\varrho_{\text{in}} \in \mathcal{S}(\mathcal{H}^S)} r\mathsf{Ent}(\varrho_{\text{in}})$.

$$\begin{array}{ccccccccc}
N\mathsf{Sep} & \subset & (N-1)\mathsf{Sep} & \subset & \cdots & \subset & 2\mathsf{Sep} & \subset & 1\mathsf{Sep} \\
\| & & & & & & \| & & \| \\
1\mathsf{Ent} & \subset & 2\mathsf{Ent} & \subset & \cdots & \subset & (N-1)\mathsf{Ent} & \subset & N\mathsf{Ent} \\
\| & & & & & & \| & & \| \\
\mathsf{EA} & & & & & & \mathsf{DGE} & & \mathsf{CPT}
\end{array}$$

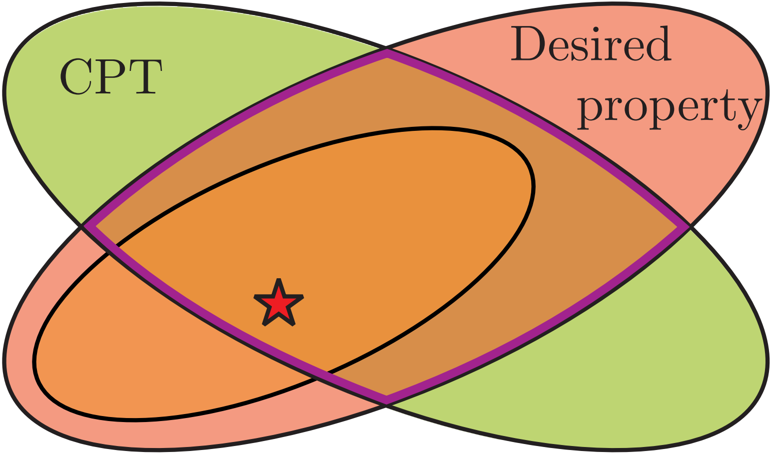

Desired
property


CPT
Desired property

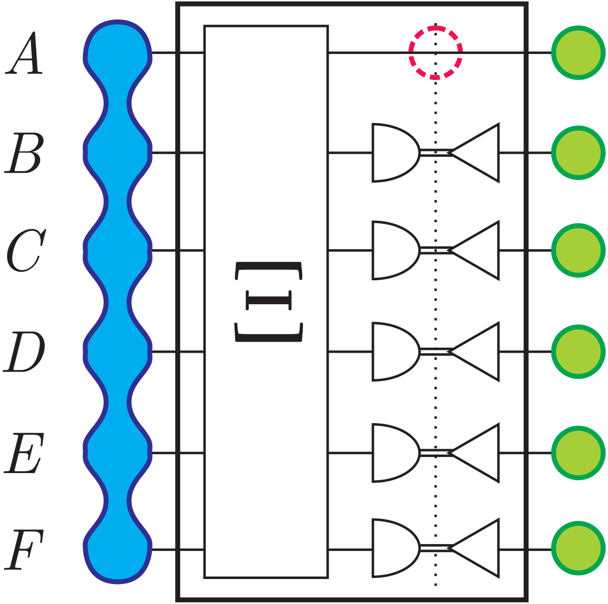

CPT
Desired property


CPT
Desired property

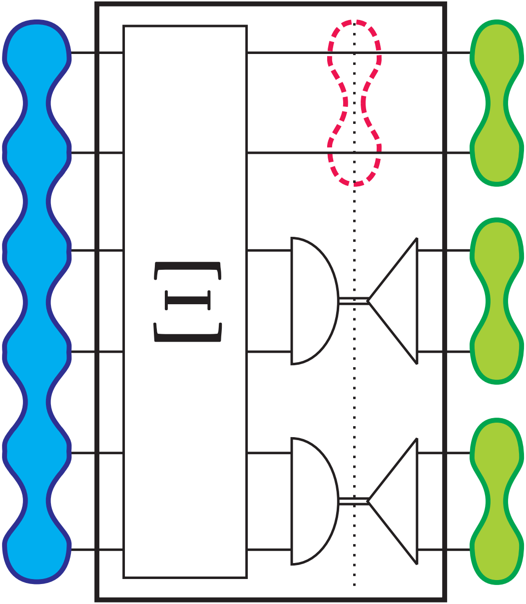
Map decomposition technique



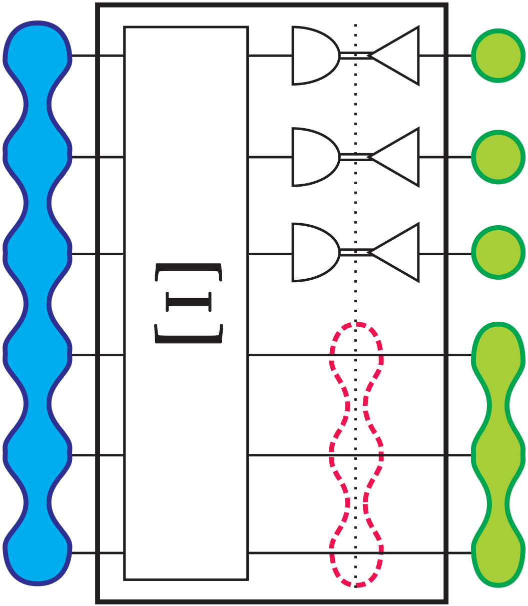
Elementary maps



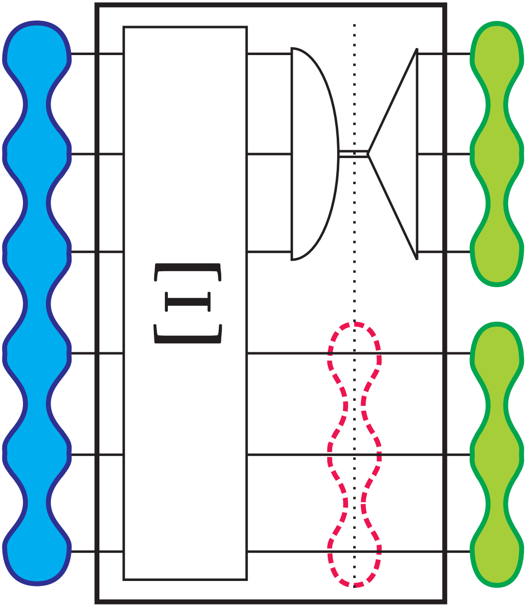
Elementary maps



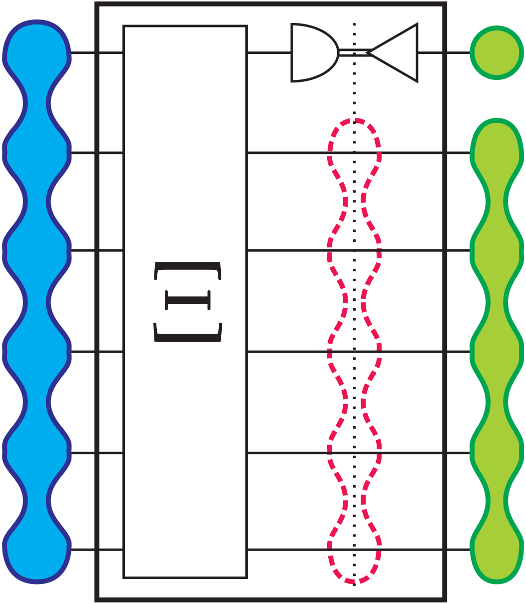
Elementary maps



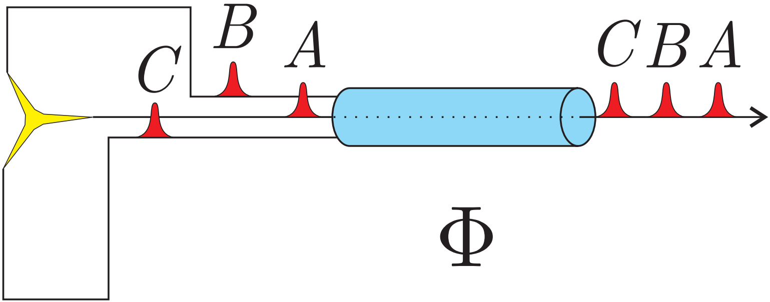
Elementary maps



Elementary maps



Local channels



$$\Phi^{\otimes N}$$

Depolarizing map $\Phi : \mathcal{T}(\mathcal{H}_d) \mapsto \mathcal{T}(\mathcal{H}_d)$ is given by

$$\Phi = q\mathrm{Id} + (1-q)\mathrm{Tr},$$

where $\mathrm{Tr}[X] = \mathrm{tr}[X]\frac{1}{d}I_d$ is the tracing map.

Results for local depolarizing noise

Local depolarizing N-qubit channel Φ_q^{local}: ranges of parameter q, for which the various entanglement-dissociative behaviors are detected (within the interval $[-\frac{1}{3}, 1]$).

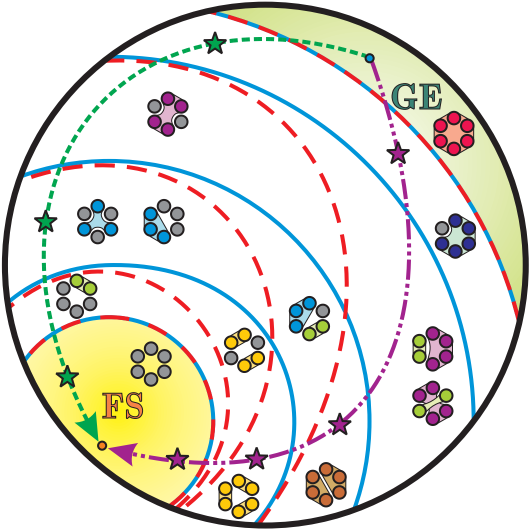
N	ϱ_{in}	EA	$\frac{N}{2}$Sep $\bigcap$ 2Ent	$(\frac{N}{2}+1)$Sep $\bigcap$ $\frac{N}{2}$ Ent	2Sep $\bigcap$ $\frac{N}{2}$ Ent	$(N-1)$Ent =DGE	Not DGE	NPT $(1, N-1)$	NPT $(\frac{N}{2}, \frac{N}{2})$
3	$\lvert GHZ\rangle$	$\leqslant$0.490	–	–	–	$\leqslant$0.713	$>$0.716[1,2]	$>$0.557	–
	$\lvert W\rangle$	$\leqslant$0.485	–	–	–	$\leqslant$0.686	$>$0.772[1]	$>$0.576	–
	ϱ_{UPB}	$\leqslant$0.698	–	–	–	$\leqslant$0.852	$\varnothing$	$\varnothing$	–
	all	$\leqslant$0.477	–	–	–	$\leqslant$0.650	–	–	–
4	$\lvert GHZ\rangle$	$\leqslant$0.453	$\leqslant$0.548	$\leqslant$0.553	$\leqslant$0.548	$\leqslant$0.751	$>$0.781[1,2]	$>$0.578	$>$0.512
	$\lvert W\rangle$	$\leqslant$0.447	$\leqslant$0.473	$\leqslant$0.581	$\leqslant$0.473	$\leqslant$0.756	$>$0.842[1]	$>$0.585	$>$0.548
	$\lvert Cl\rangle$	$\leqslant$0.444	$\leqslant$0.478	$\leqslant$0.574	$\leqslant$0.478	$\leqslant$0.742	$>$0.774[1]	$>$0.532	$>$0.550
	all	$\leqslant$0.444	$\leqslant$0.472	$\leqslant$0.550	$\leqslant$0.472	$\leqslant$0.715	–	–	–
6	$\lvert GHZ\rangle$	$\leqslant$0.414	$\leqslant$0.433	$\leqslant$0.591	$\leqslant$0.530	$\leqslant$0.826	$>$0.850[2]	$>$0.638	$>$0.490

[1] Computation via the method of Jungnitsch et al. (2011)

[2] Computation via the method of Seevinck et al. (2008)

Local vs. global noises

$$q \sim e^{-\Gamma t}$$



Continuous-variable systems

States

Characteristic function $\varphi(\mathbf{z}) = \mathrm{tr}[\varrho W(\mathbf{z})]$

- Weyl operator
  $W(\mathbf{z}) = \exp[i(q_1 x_1 + p_1 y_1 + \cdots + q_N x_N + p_N y_N)]$
- number of modes $\quad N$
- canonical commutation relation $\quad [q_i, p_j] = i\delta_{ij}$
- coordinates in the real symplectic space
  $\mathbf{z} = (x_1, y_1, \ldots, x_N, y_N)^\top$
- symplectic form $\quad \mathbf{\Delta} = \bigoplus_{i=1}^{N} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Attenuator and amplifier

$$\varphi_{\text{out}}(\mathbf{z}) = \varphi_{\text{in}}(\mathbf{K}\mathbf{z}) \exp\left(-\frac{1}{2}\mathbf{z}^\top \mathbf{M}\mathbf{z}\right)$$

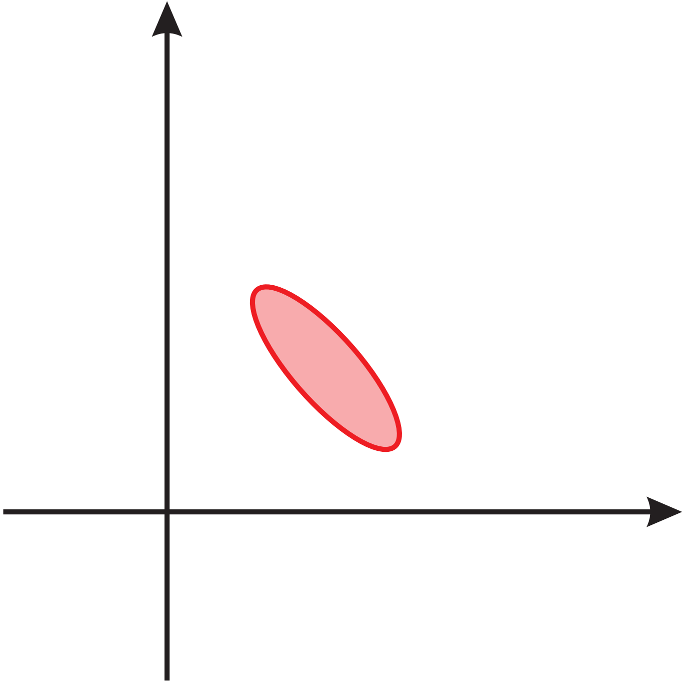
$$\mathbf{K} = \sqrt{\kappa}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{M} = \mu\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

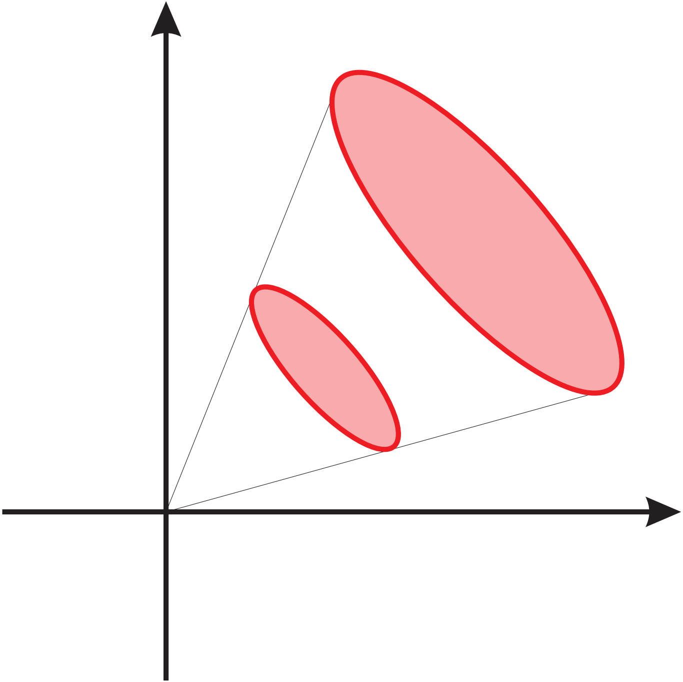
- attenuation $(0 < \kappa < 1)$
- addition of classical noise $(\kappa = 1)$
- amplification $(\kappa > 1)$

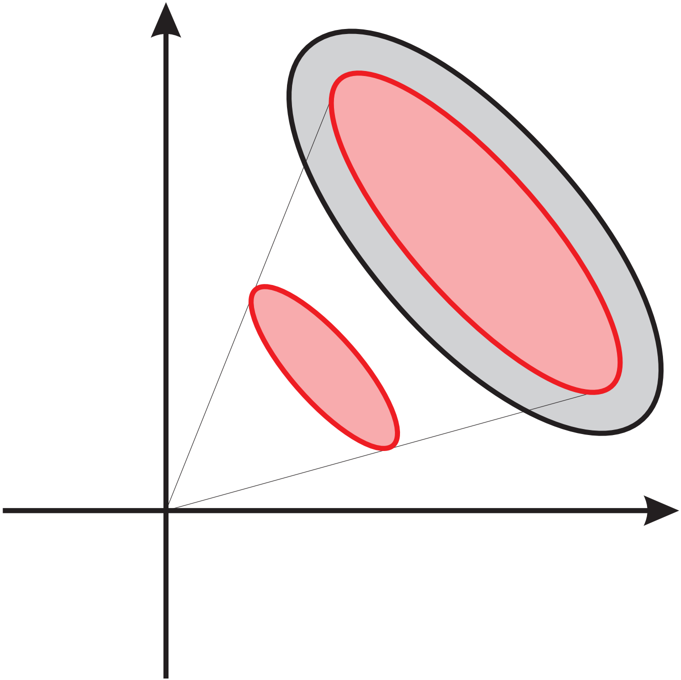
Fair physical channels (completely positive maps): $\mu \geq \frac{1}{2}|\kappa - 1|$

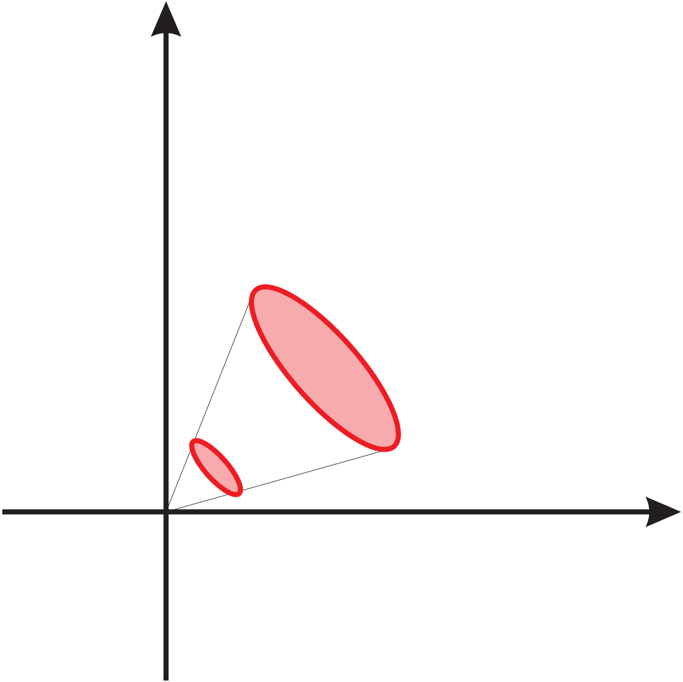
Quantum limited operation: $\mu_{\text{QL}} = \frac{1}{2}|\kappa - 1|$

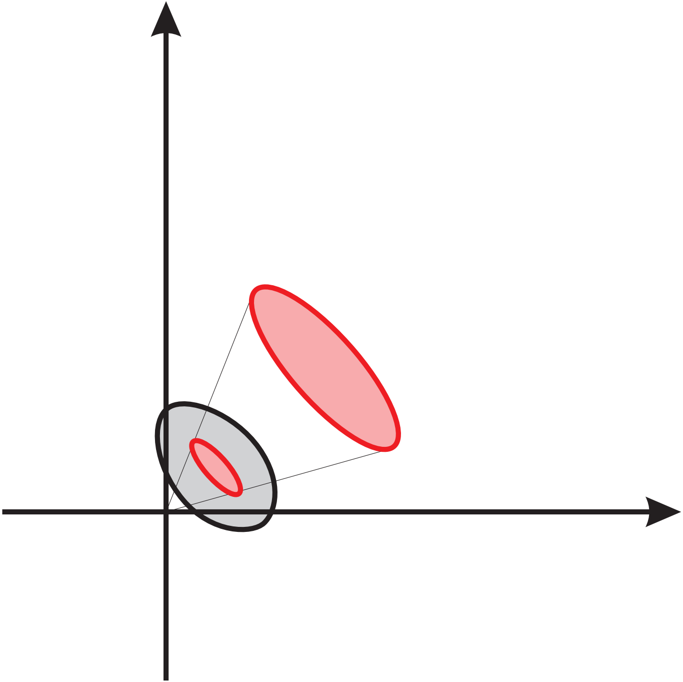
Extra noise: $a = \mu - \mu_{\text{QL}} \geq 0$



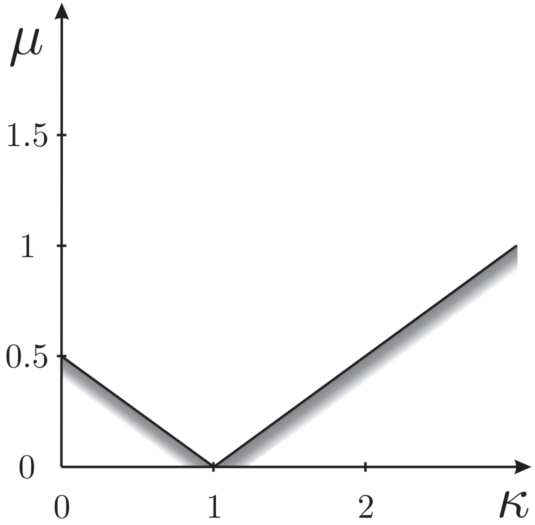




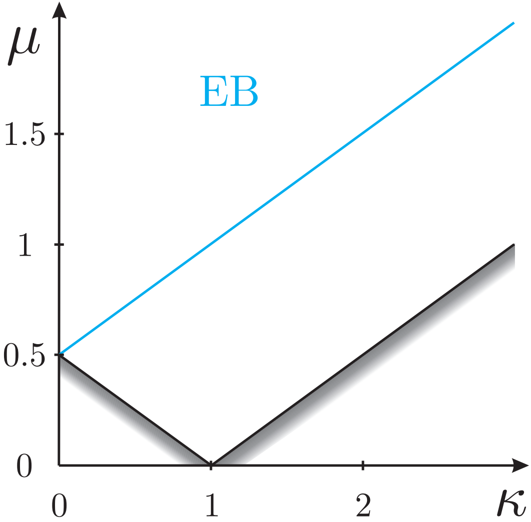




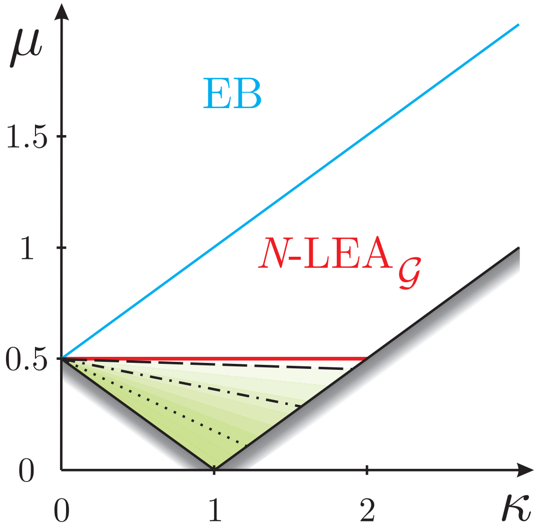
Attenuator and amplifier



Attenuator and amplifier

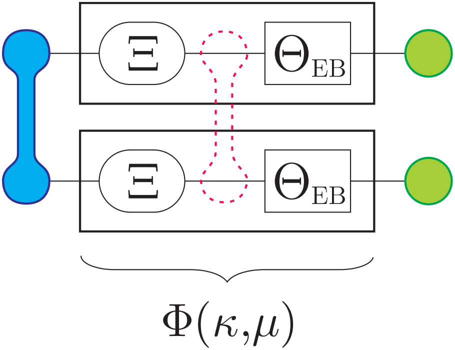


N-local attenuator and amplifier: Gaussian input



Proposition. The channel $\Phi(\kappa, \mu)$ is N-LEA$_{\mathcal{G}}$ if and only if the total noise level $\mu \geq \frac{1}{2}$.

Proof.



Φ is N-LEA if $\Phi^{\otimes N}$ annihilates entanglement

$$\mathsf{EB}^{\mathcal{C}} \subsetneq \infty\text{-}\mathsf{LEA}^{\mathcal{C}}_{\mathcal{G}} = \cdots = 3\text{-}\mathsf{LEA}^{\mathcal{C}}_{\mathcal{G}} = 2\text{-}\mathsf{LEA}^{\mathcal{C}}_{\mathcal{G}}$$

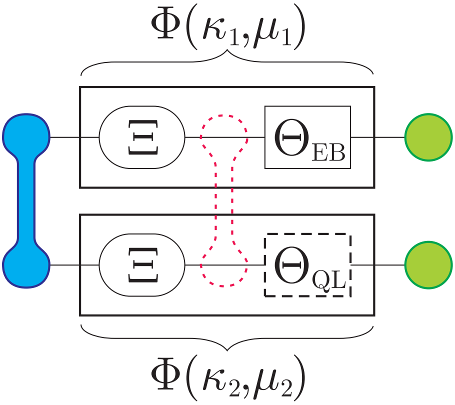
cf.

$$\mathsf{EB} \subset \infty\text{-}\mathsf{LEA} \subset \cdots \subset 3\text{-}\mathsf{LEA} \subset 2\text{-}\mathsf{LEA}$$

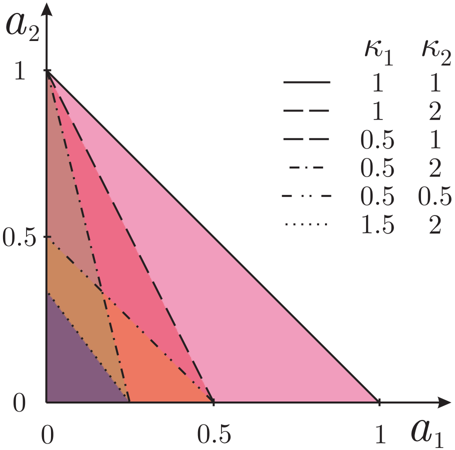
General local action on two-mode gaussian inputs

Proposition. The channel $\Phi(\kappa_1, \mu_1) \otimes \Phi(\kappa_2, \mu_2)$ annihilates entanglement of all two-mode gaussian states if and only if $\kappa_1\mu_2 + \kappa_2\mu_1 \geq (\kappa_1 + \kappa_2)/2$.

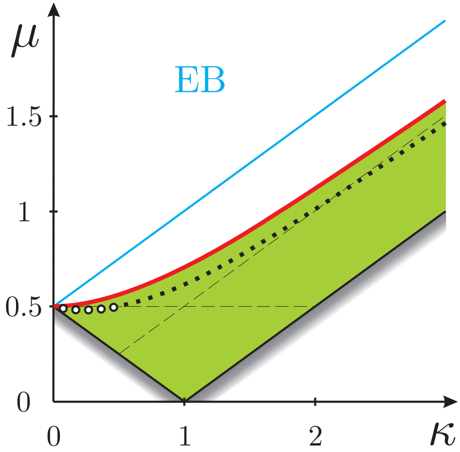
Proof.



General local action on two-mode gaussian inputs



2-local attenuator and amplifier: Non-gaussian inputs



Corollary. The channel $\Phi(\kappa, \mu) \in \mathcal{C}$ is not N-LEA for any $N = 2, 3, \ldots$ if the total noise level satisfies $\mu < \frac{1}{2}\sqrt{\kappa^2 + 1}$.

Summary

- Criteria for maps dissociating entanglement
- Map decomposition technique
- ... which enables to consider *all* input states
- Differences in entanglement dynamics under local and global noises
- Attenuation and amplification of signals carrying gaussian and non-gaussian entanglement

Thank you for listening!

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