Distance to boundary and minimum-error discrimination

Erkka Haapasalo, Michal Sedlák, and Mário Ziman

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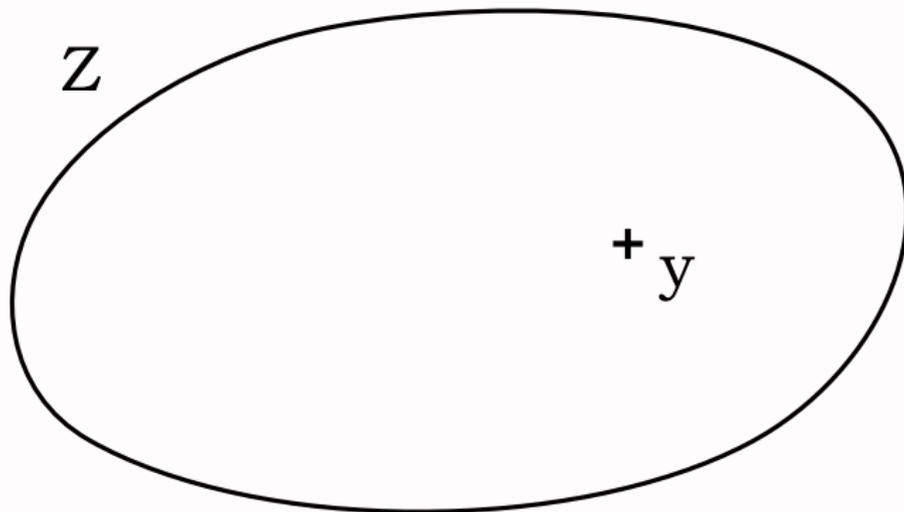
Generalities

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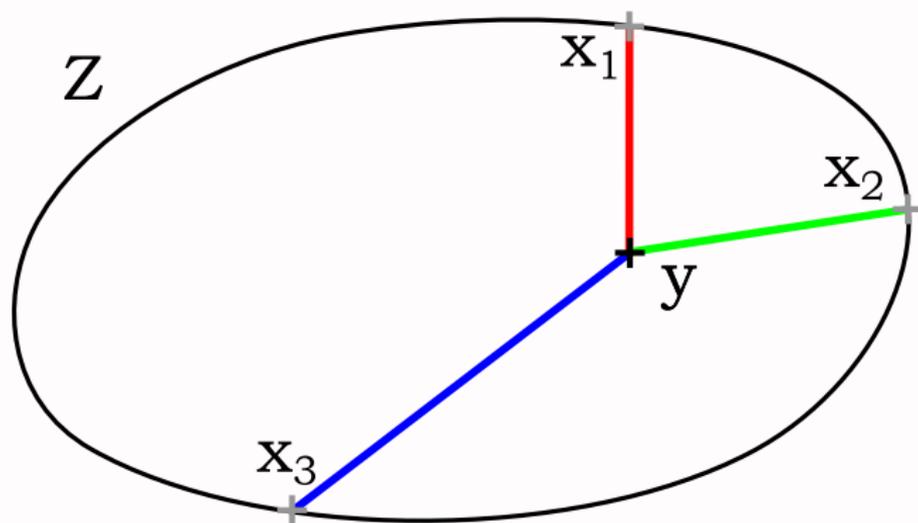
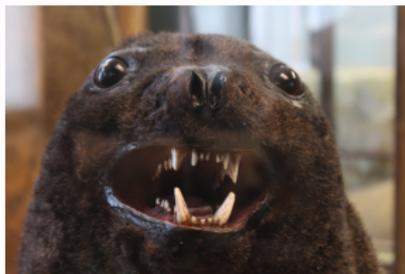# Boundariness

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We use the convex structure to measure distance.

$b(y) = 1 - w(y)$
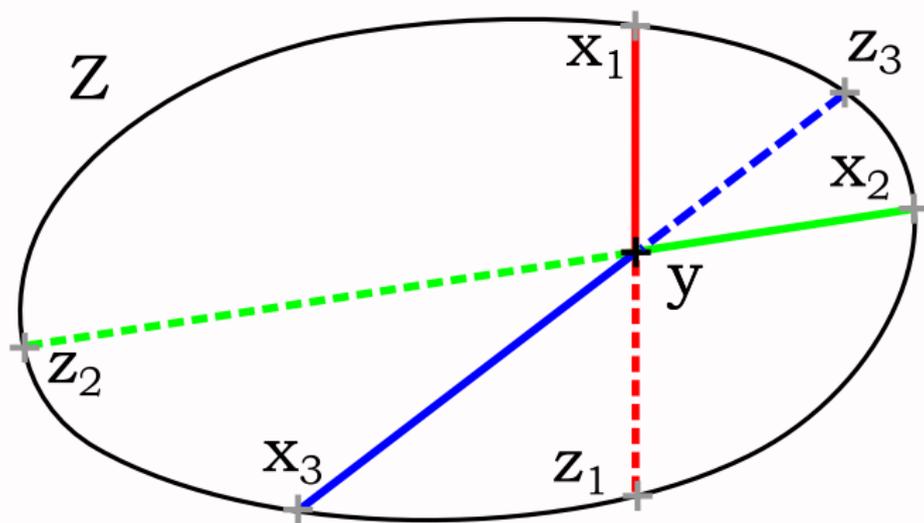
$w(y) =$ the highest weight with which any boundary element can appear in a convex decomposition of y.

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Proposition

Suppose that $Z \subset \mathbb{R}^n$, $n = 1, 2, \ldots$, is compact and convex and $y \in Z \setminus \partial Z$. There is an extreme point x of Z and $z \in \partial Z$ such that $t_y(x) = b(y)$ and

$$y = b(y)x + \big(1 - b(y)\big)z.$$
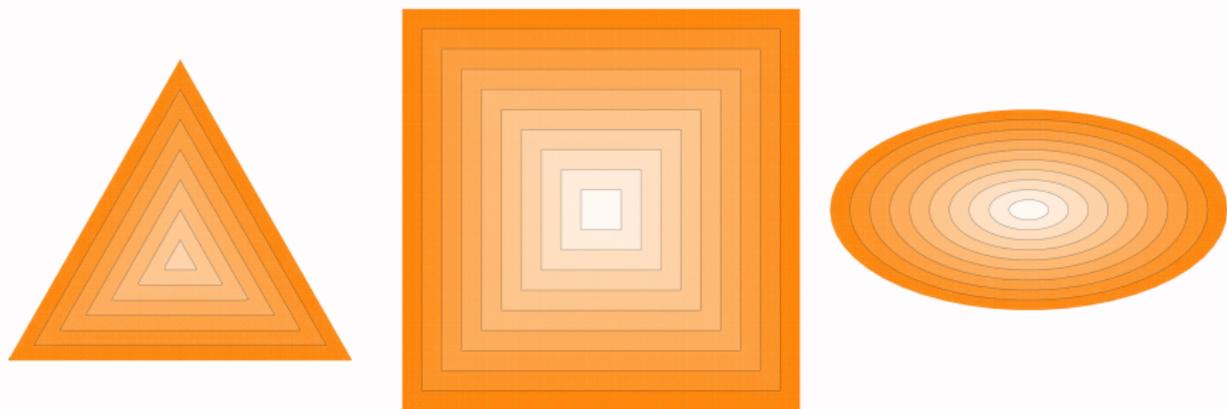
Boundariness plotted for a few simple convex plane sets.

Boundariness and bounded seminorms

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For any x, $y \in Z$, one has

$$p(x - y) \leq 2a\big(1 - b(y)\big).$$

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In this context we can define the base norm $\| \cdot \|_Z$ generated by Z:

$$\|v\|_Z = \inf_{\lambda, \mu \geq 0} \{\lambda + \mu \,|\, v = \lambda x - \mu y, \ x, \, y \in Z\}, \quad v \in V.$$

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  $\sup_{x \in Z} \|x - y\|_Z \leq 2\big(1 - b(y)\big)$ for all $y \in Z$.
- Base norms are directly linked to quantum discrimination problems. $\Rightarrow$ Boundariness is connected to quantum discrimination.

Boundariness of quantum devices

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- Set of discrete N-outcome observables $\mathcal{O}_\Omega(\mathcal{H})$: POVMs $\mathsf{A} = (\mathsf{A}_j)_{j=1}^N$, $\mathsf{A}_j \geq 0$, $\sum_{j=1}^N \mathsf{A}_j = \mathbb{1}_\mathcal{H}$.

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- A channel $\mathcal{E} \in \mathcal{C}(\mathcal{H}, \mathcal{K})$ describes a transformation of a system associated with $\mathcal{H}$ into a system associated with $\mathcal{K}$.

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- The choice of the norm depends on the type of quantum devices being considered (states: the trace norm, observables: the diamond norm $\|\mathsf{M}\| = \sum_j \|\mathsf{M}_j\|$).

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- The best distiguishable quantum devices reside on the boundary.
- If $\|A - B\| = 2(1 - b(A))$ for some B, then A is best discriminable from B and $p_{\text{error}}(A, B) = \frac{1}{2}b(A)$.

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Proposition

Suppose that $\rho \in \mathcal{S}(\mathcal{H})$. There is a (pure) state $\sigma \in \mathcal{S}(\mathcal{H})$ that is the best discriminable from ρ, i.e.,

$$p_{\mathrm{error}}(\rho, \sigma) = \frac{1}{2}b(\rho).$$

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- For each M there exists a best discriminable A (which is extreme, even projection valued), i.e., $p_{\mathrm{error}}(\mathsf{M}, \mathsf{A}) = \frac{1}{2} b(\mathsf{M})$.

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We do not know yet. . .

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$\mathcal{E}_p(\rho) = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$ for all qubit states ρ.

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$\Rightarrow$ We conjecture: For each channel $\mathcal{E}$ there is a best discriminable extreme (maybe even unitary) channel $\mathcal{F}$ with $p_{\mathrm{error}}(\mathcal{E}, \mathcal{F}) = \frac{1}{2}b(\mathcal{E})$.

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- Does boundariness give a strict lower bound the for minimum error probability in channel discrimination?
- These questions have been answered for the qubit "erasure" channels and for any channel such that the $\lambda_{\min}$-eigenspace contains a maximally entangled vector.

References

- E.H., M. Sedlák, and M. Ziman, arXiv:1401.7460

- Further reading:

- A. Jenčová, J. Math. Phys. 55, 022201 (2014)

- G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A 80, 022339 (2009)

- G. M. D'Ariano, P. Lo Presti, and P. Perinotti, J. Phys. A: Math. Gen. 38, 5979 (2005)

- D. Reeb, M. J. Kastroyano and M. M.Wolf, J. Math. Phys. 52, 082201 (2011)

Thank you for your attention!