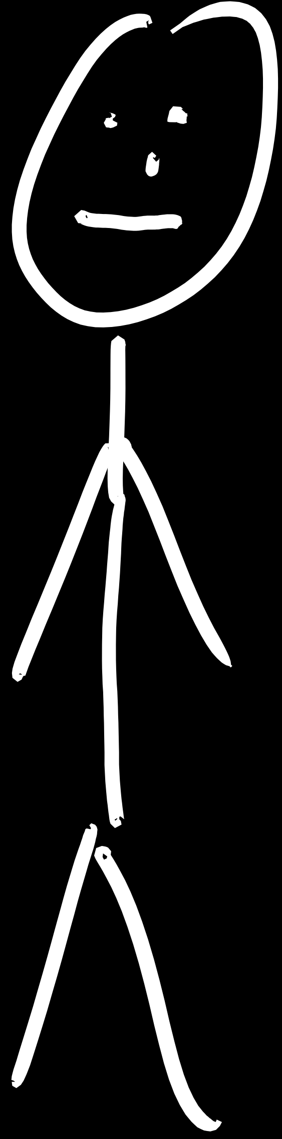
Process estimation in pressence of memory

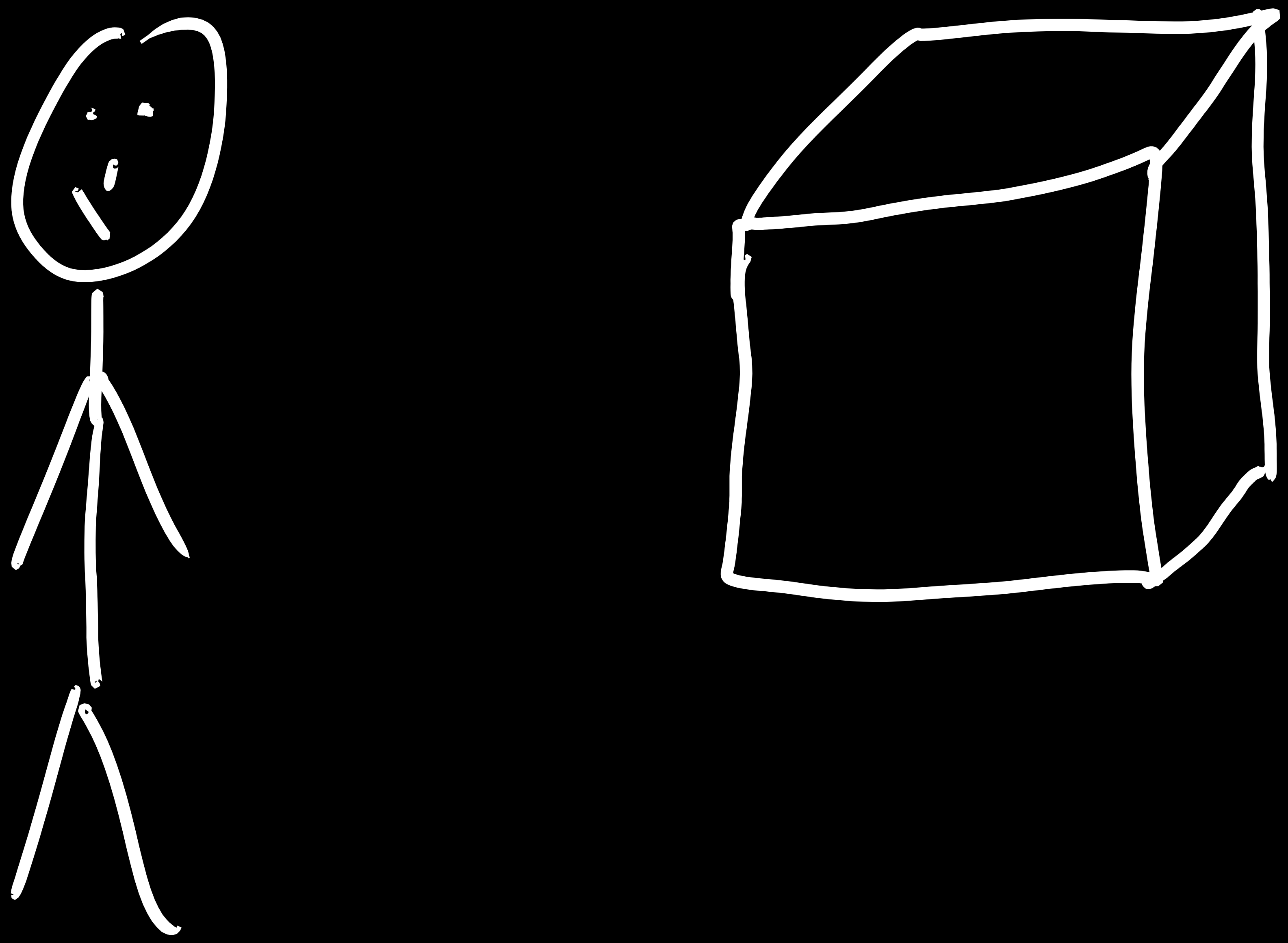
T.R. & Mário Ziman

CEQIP 2014

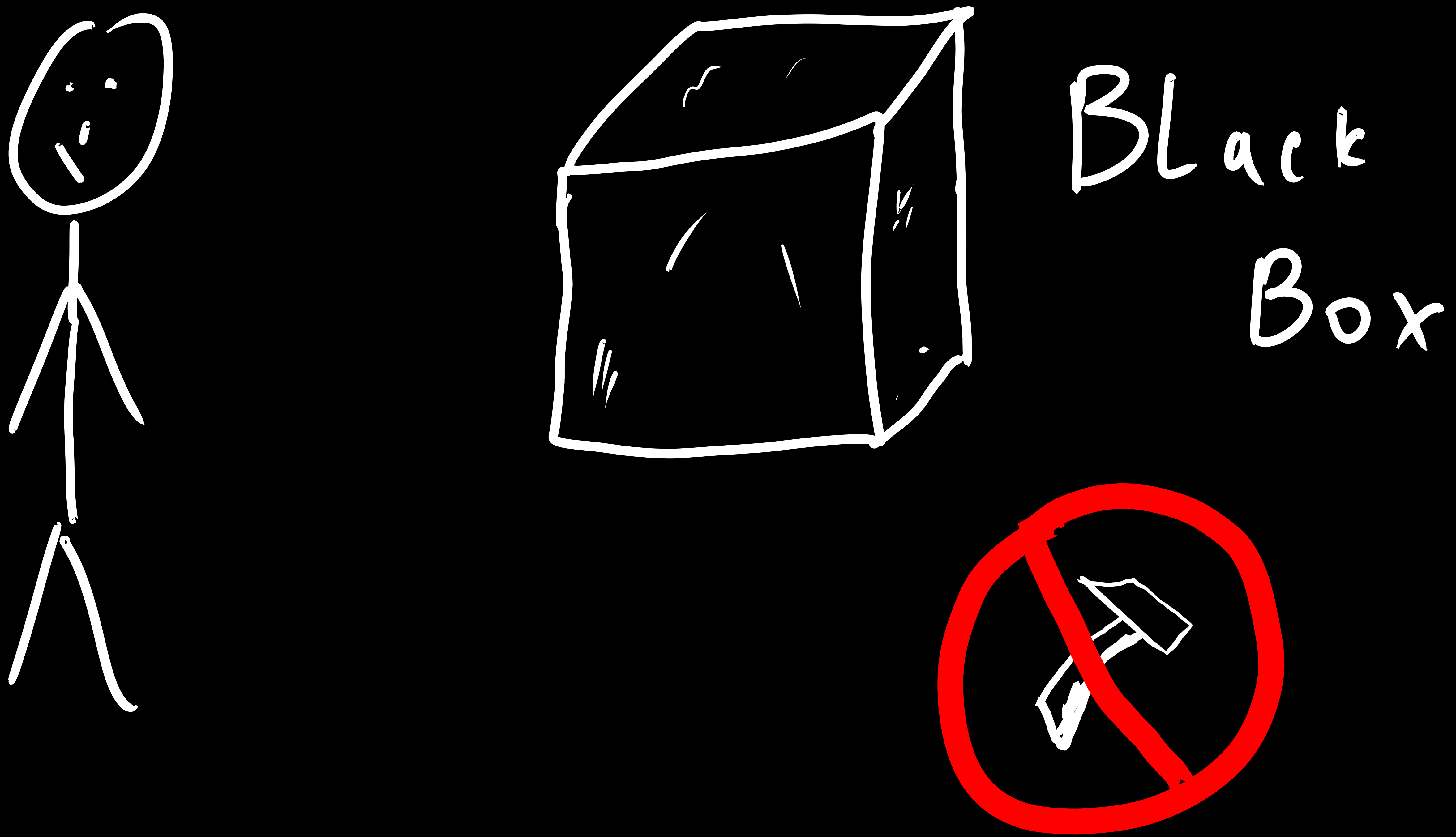


Experimenter

(Honest)



Black
Box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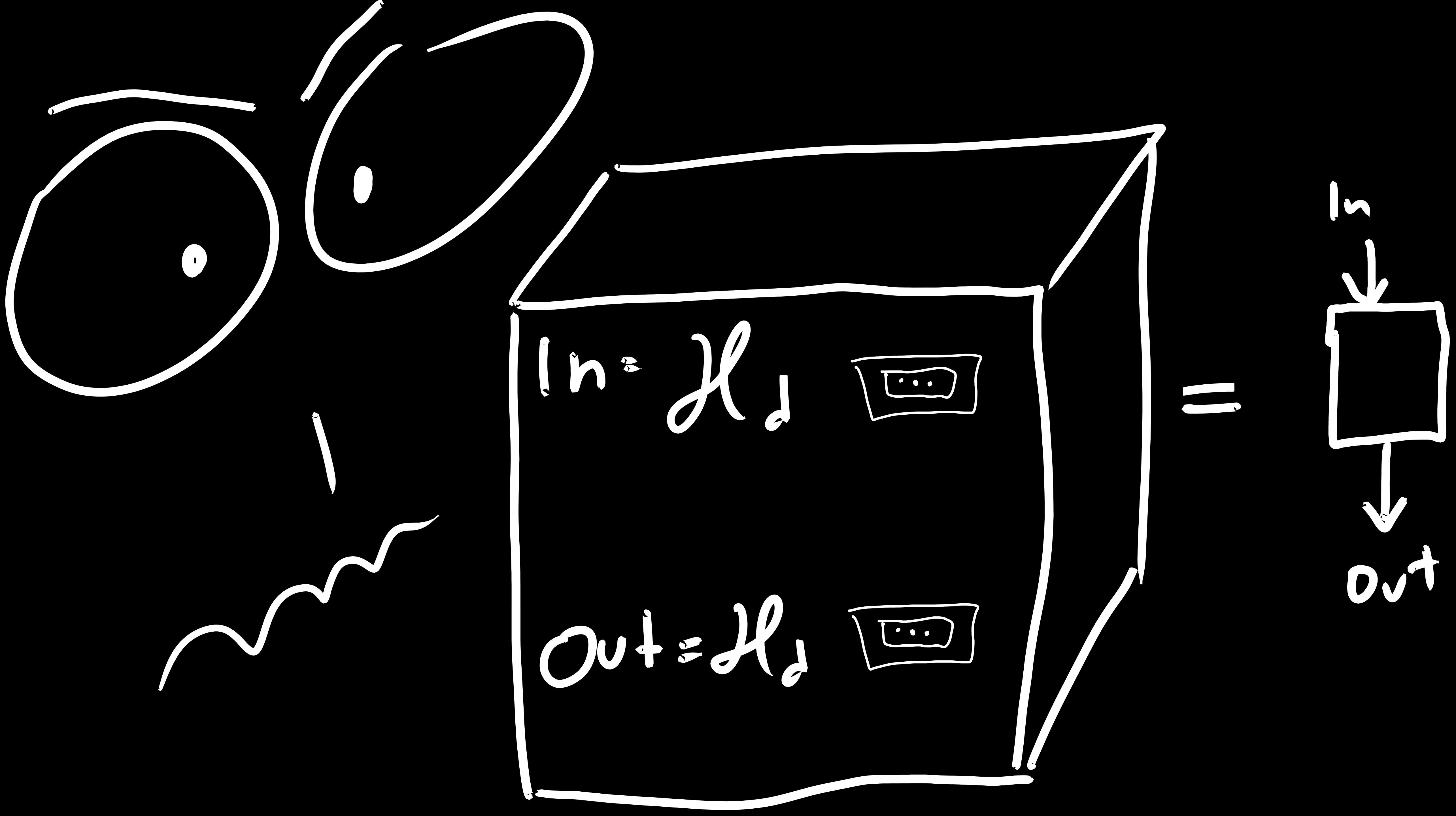
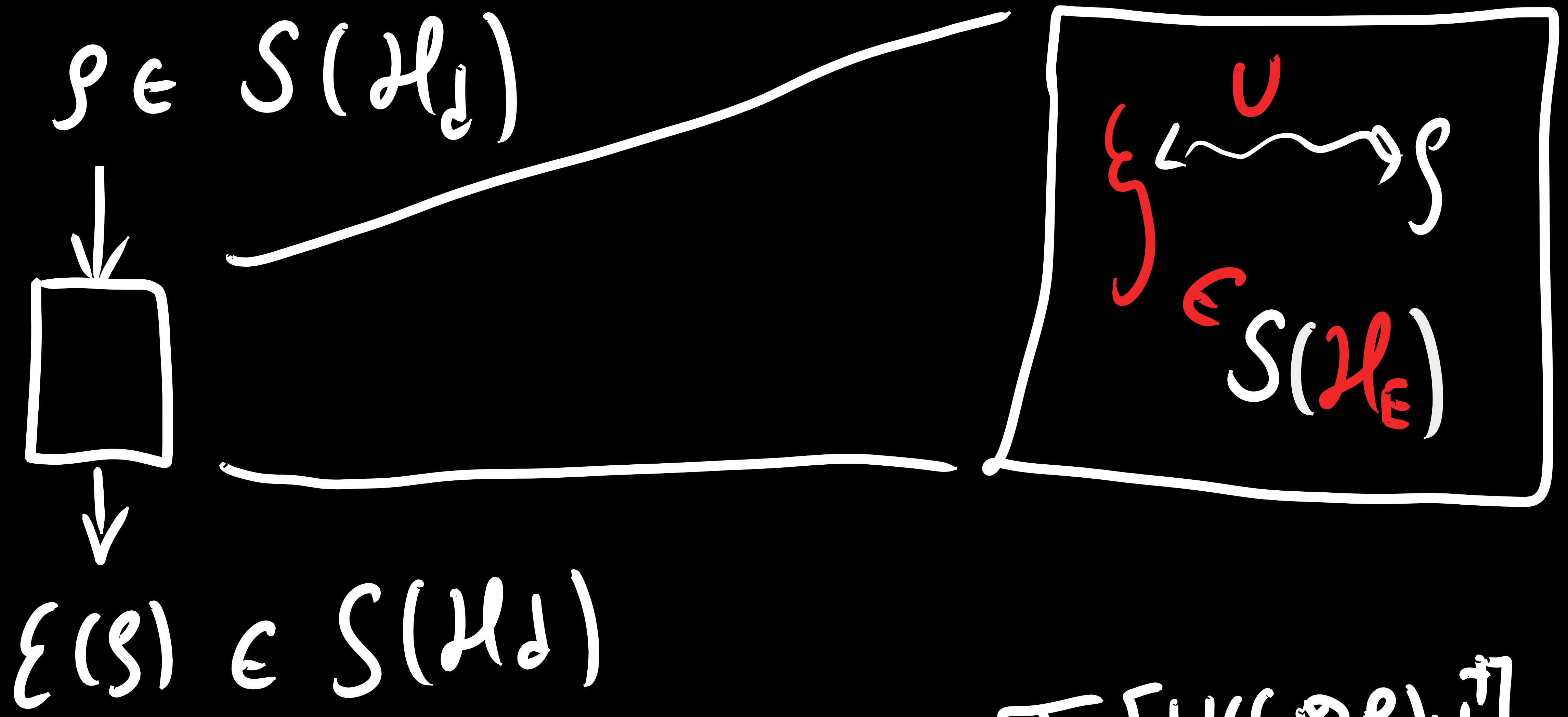
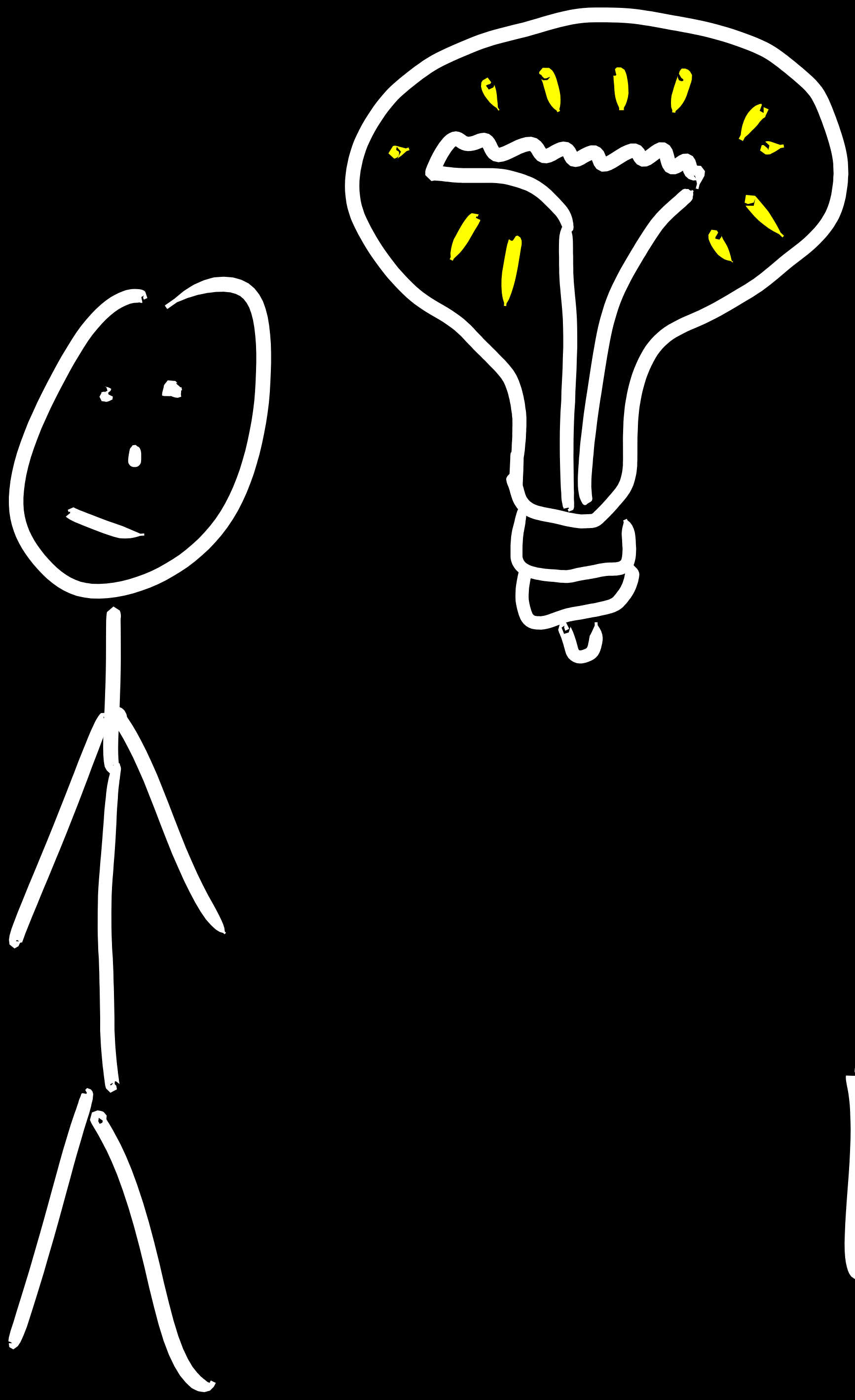
RTFM!

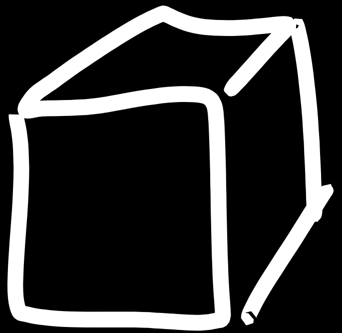

In: $\mathcal{H}_d$
Out = $\mathcal{H}_d$
In
Out



$$\mathcal{E}(\rho) = \mathrm{Tr}_E\left[U(\xi \otimes \rho)U^\dagger\right]$$

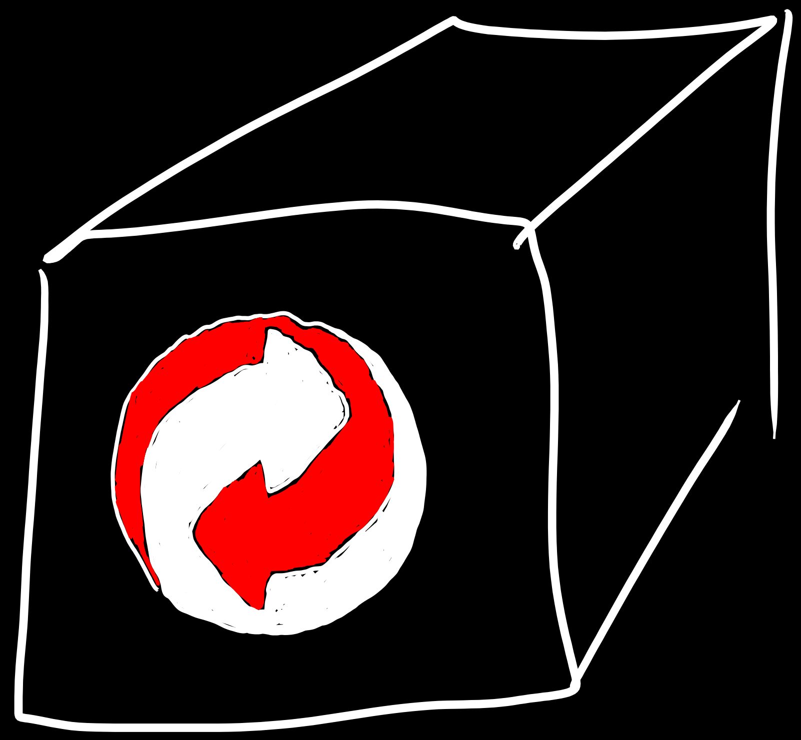


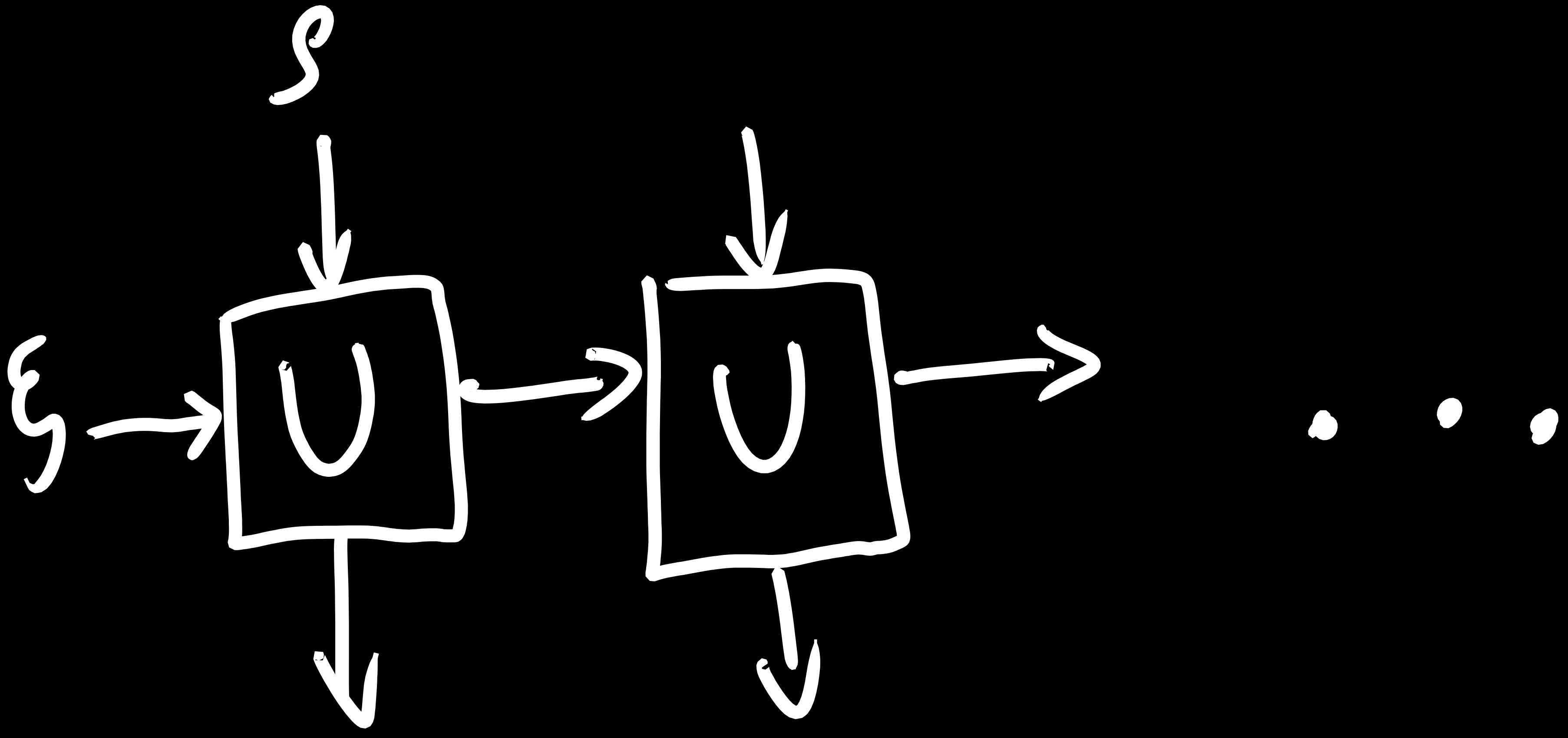
Process Tomography



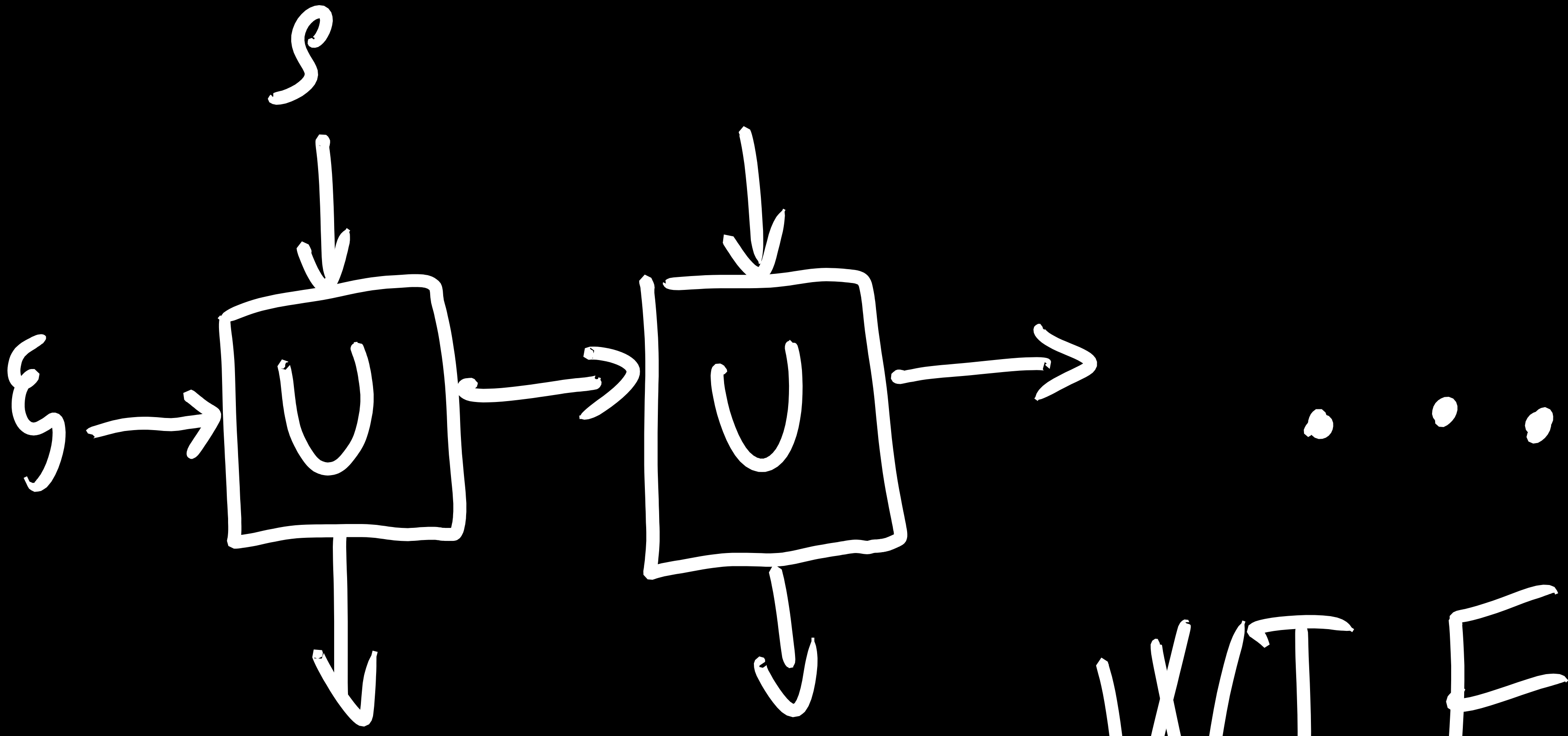
$\longrightarrow \mathcal{E}$

REUSE!



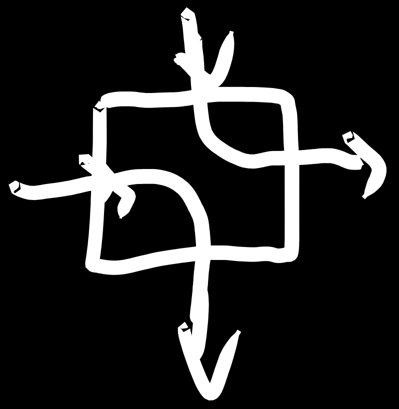


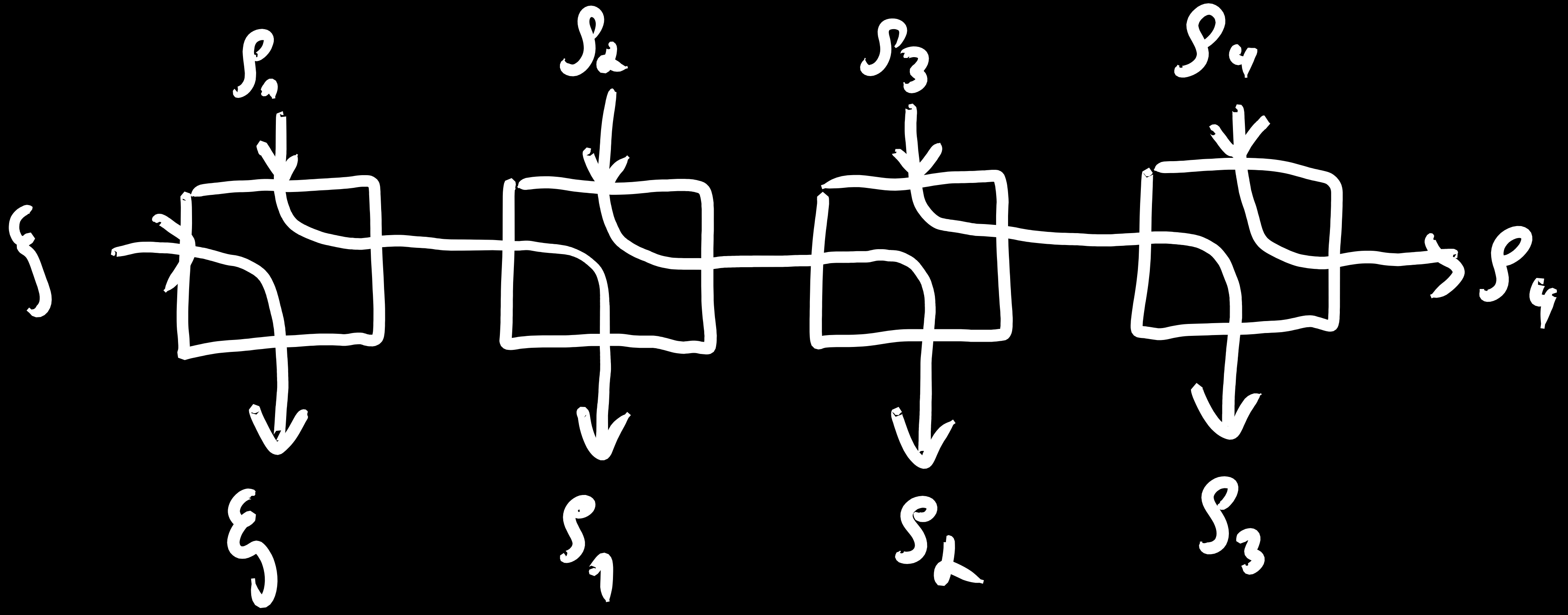
Memory Channel



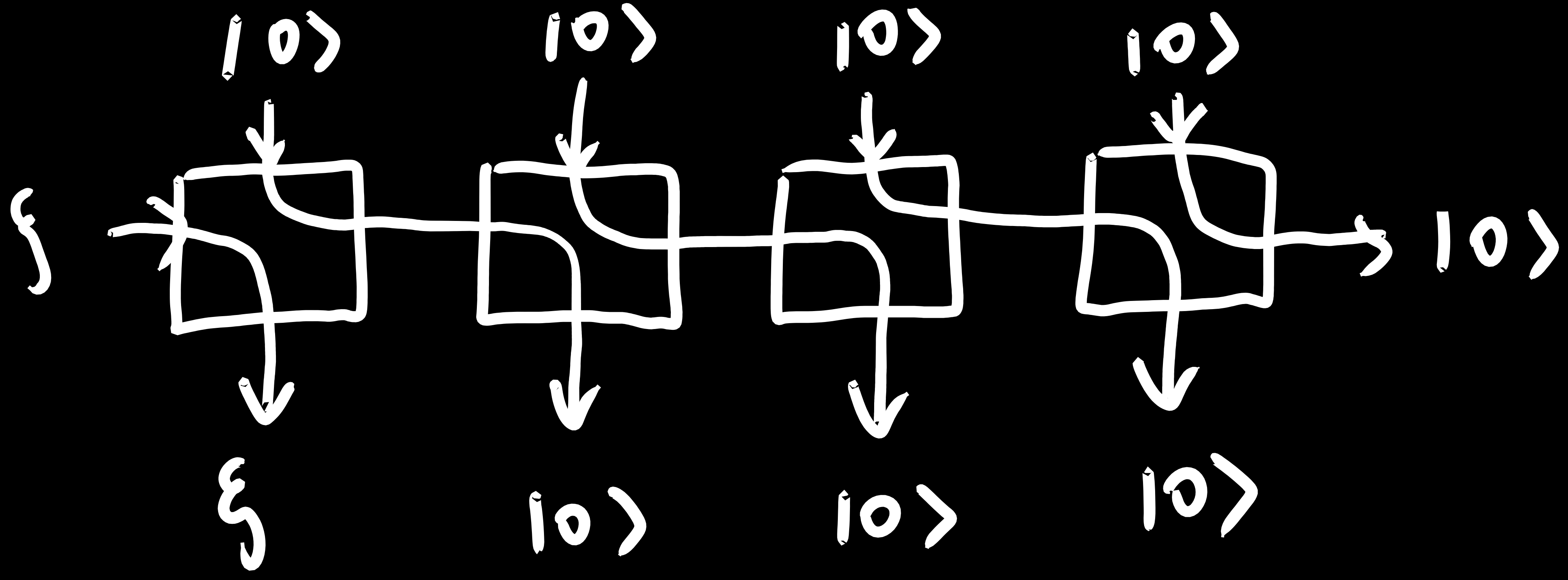
WTF is Process Tomography now?!

$U = \mathrm{SWAP}$ $\qquad$ $U(A \otimes B)U^{\dagger} = B \otimes A$





$$\rho_i \longrightarrow \rho_{i-1}$$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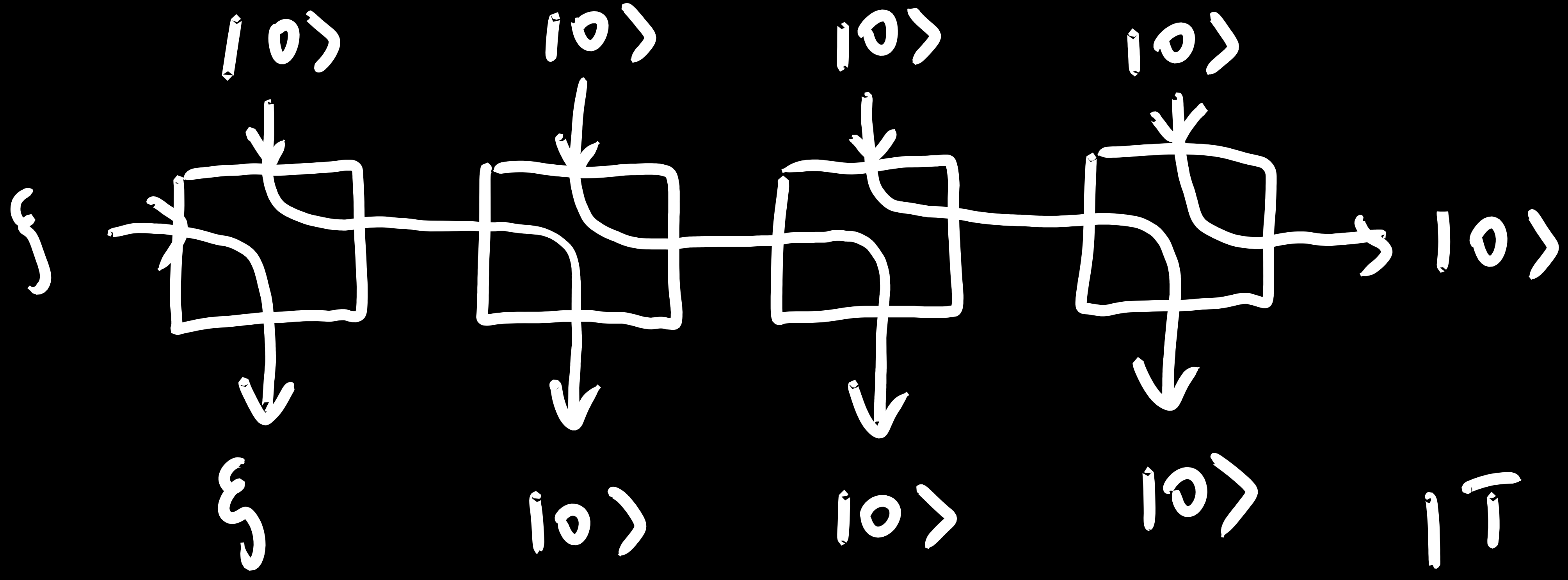
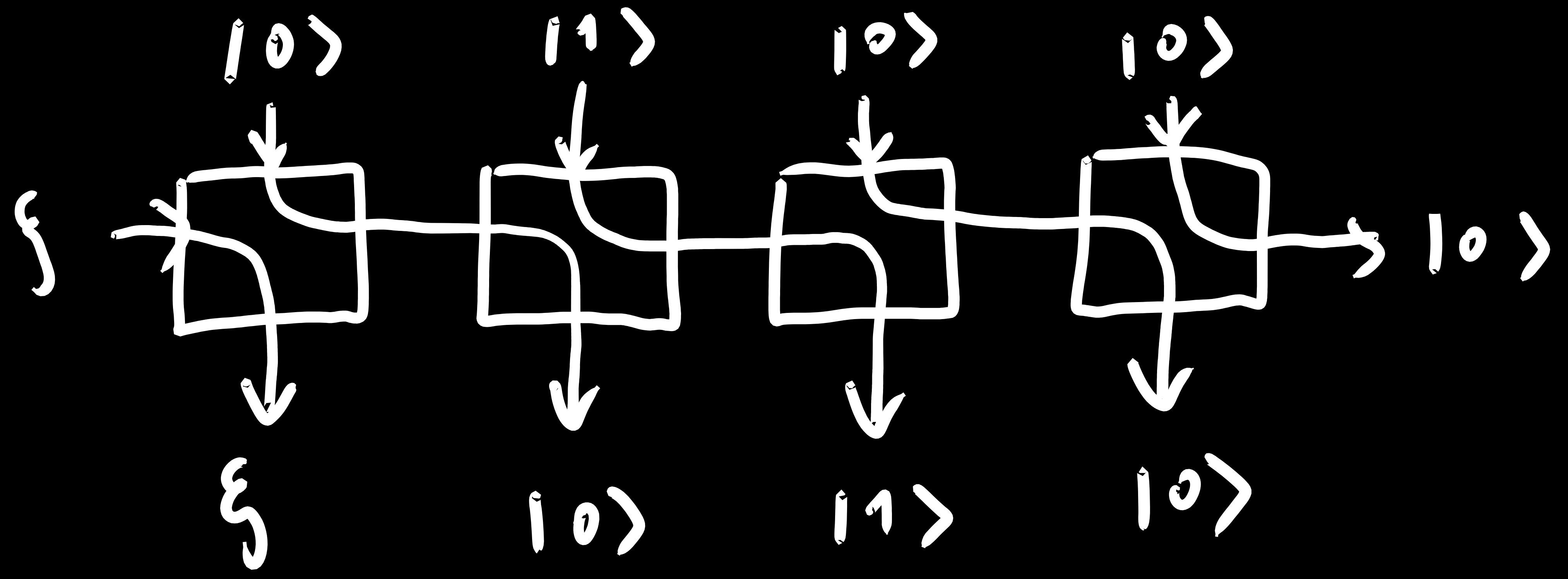


IT is a TRIVIAL channel!

$$|n\rangle \to |n\rangle$$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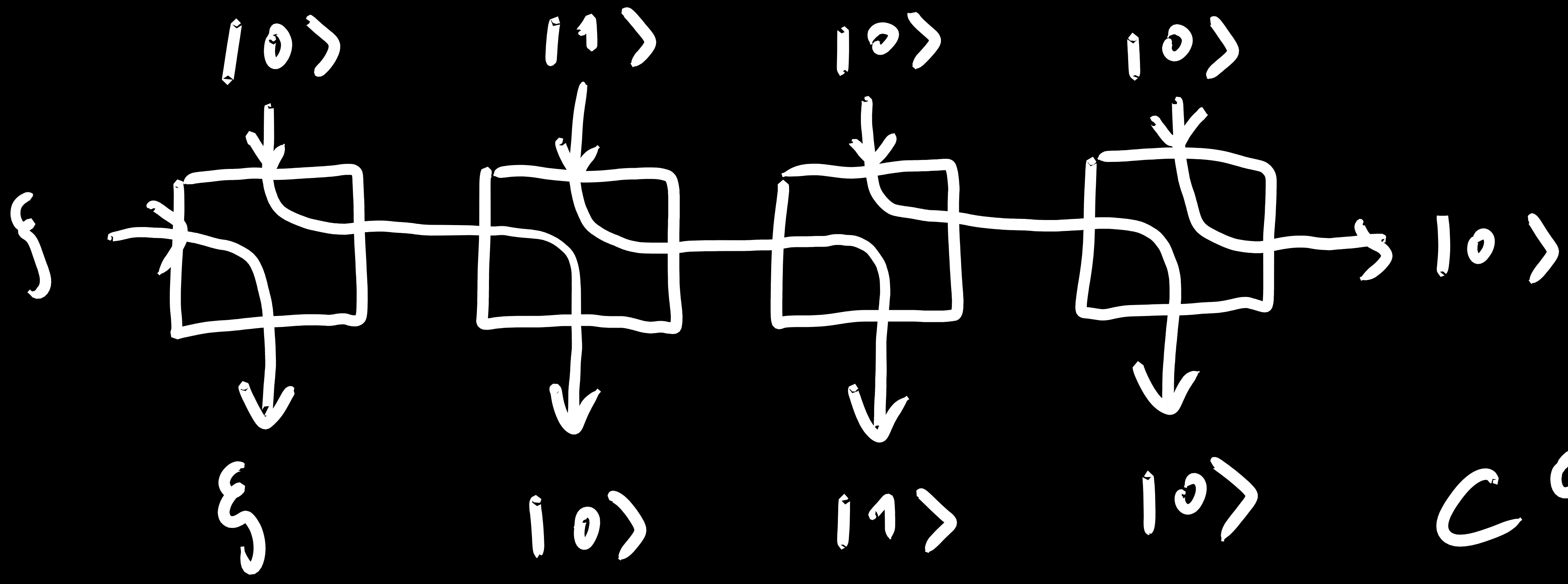
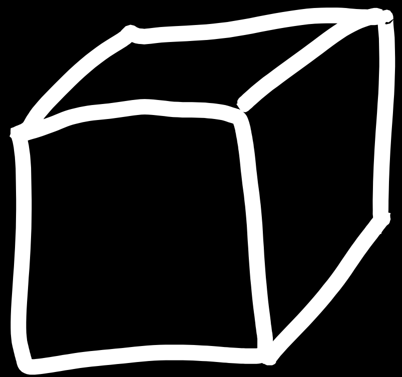


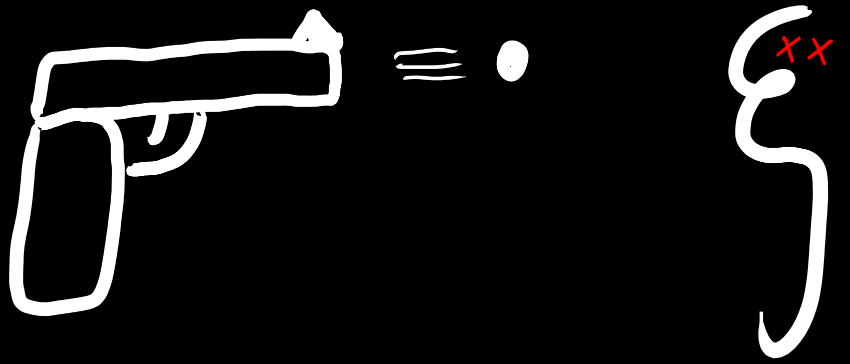
$$|n\rangle \rightsquigarrow \frac{1}{d}\mathbb{1}$$

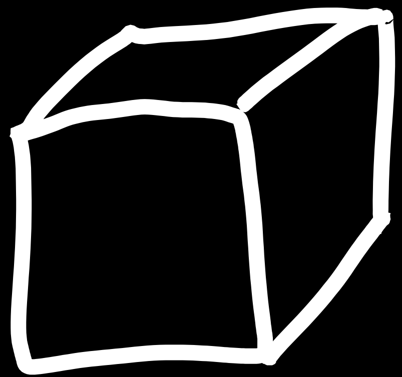
CONTRACTION To maximal mixture!

$\cancel{\varepsilon}$



$\longrightarrow (U, \xi)$



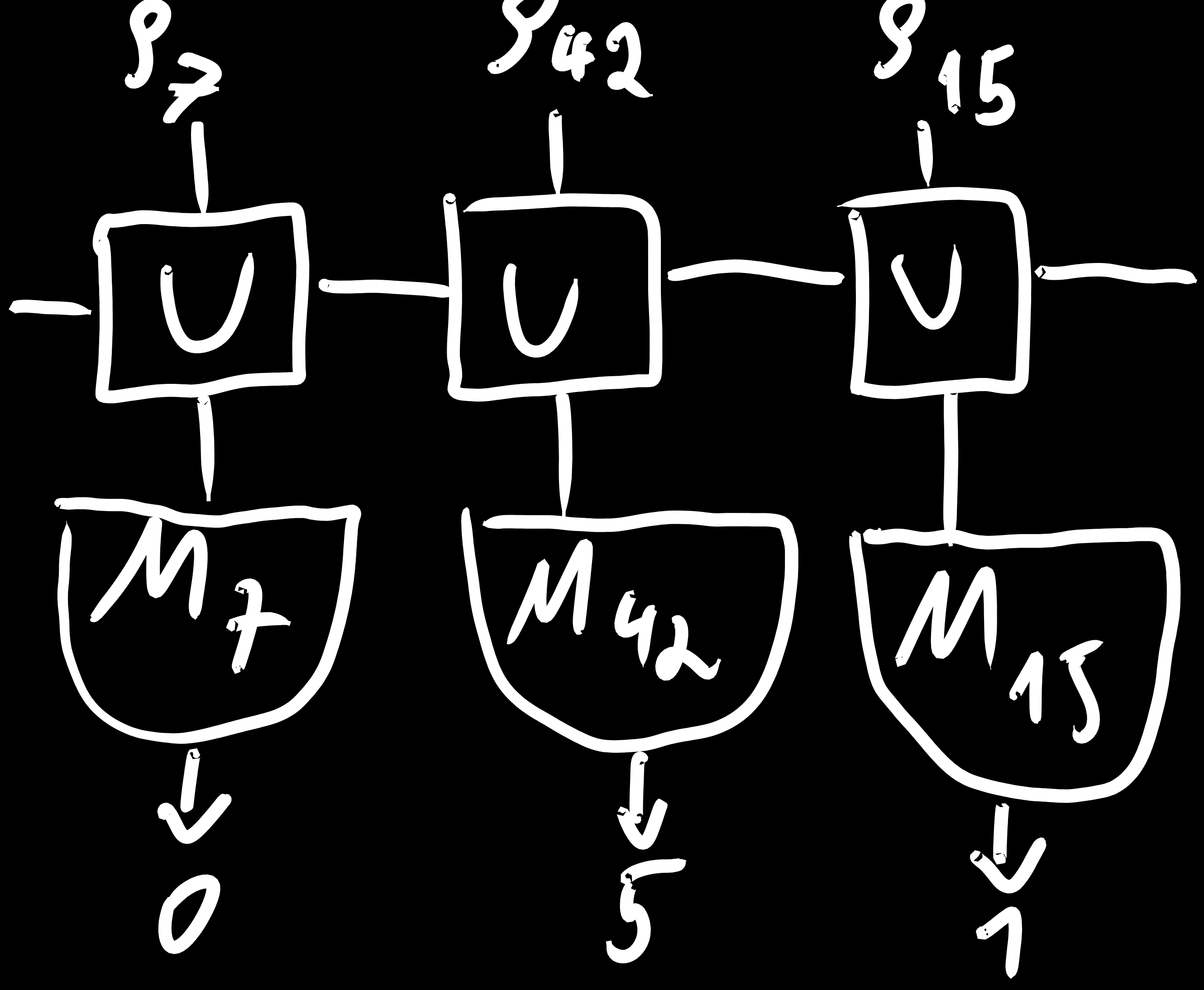


$\longrightarrow (U, \cancel{\xi})$

How to get U?

How to get ♥U?

actually – Standard
Process
Tomography
+ Random order



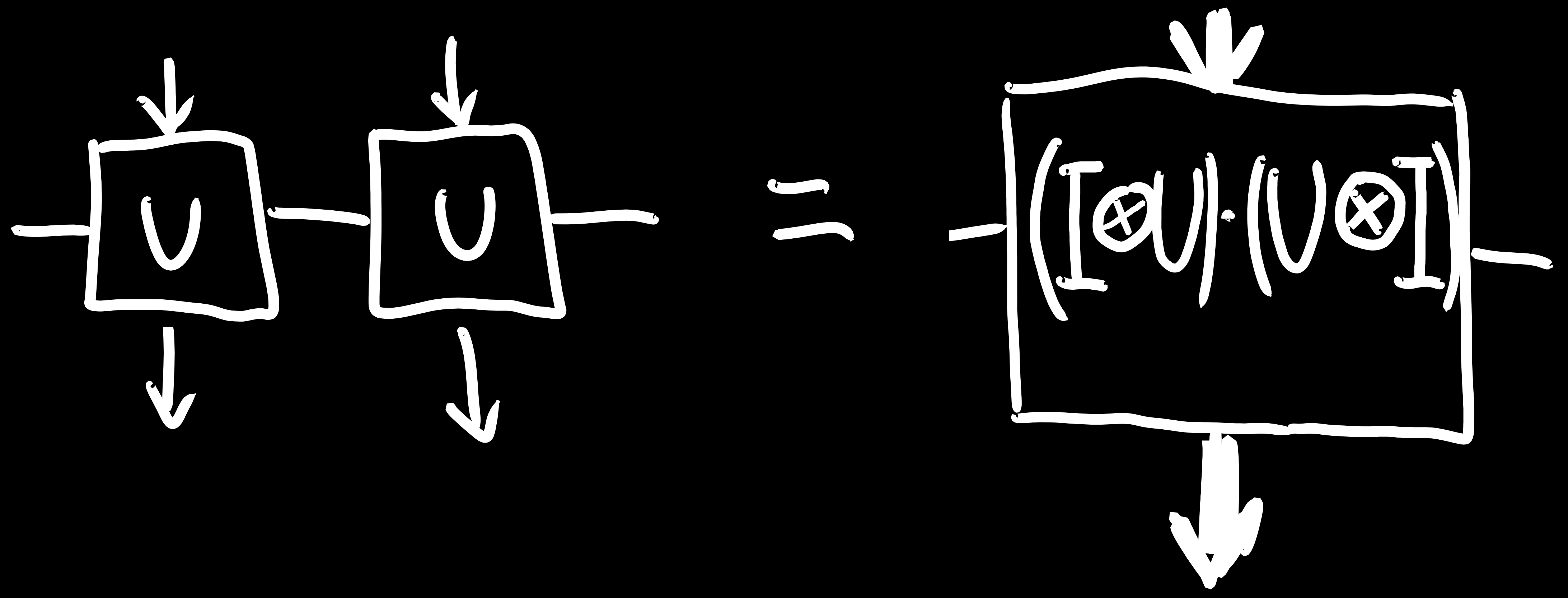
$$(\rho_1, M_1, p_1)$$
$$(\rho_2, M_2, p_2)$$
$$\vdots$$
$$(\rho_N, M_N, p_N)$$

$$\sum_i p_i = 1$$

Then $\exists\, \bar{\xi} \in S(\mathcal{H}_E)$

$$\mathrm{Prob}(M_{ij} | \rho_i) = \mathrm{Tr}\left[U(\bar{\xi} \otimes \rho_i)U^\dagger(I \otimes M_{ij})\right]$$

$$\iff \quad \mathcal{E}_1(\rho) = \mathrm{Tr}_E\left[U(\bar{\xi} \otimes \rho)U^\dagger\right]$$



$$\mathcal{E}_2(\rho\otimes\sigma) = \mathrm{Tr}_E\left[U_2(\bar{\xi}\otimes\rho\otimes\sigma)U_2^\dagger\right]$$

$\dots$

$$\varepsilon_1$$

$$\varepsilon_2 \xrightarrow{\text{(Max Lik, AIC)}} U, \bar{\xi}$$

$$\vdots$$

$$\varepsilon_N$$

$$\begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{matrix} \xrightarrow{(\text{Max Lik, AIC})} U, \bar{\varepsilon}$$

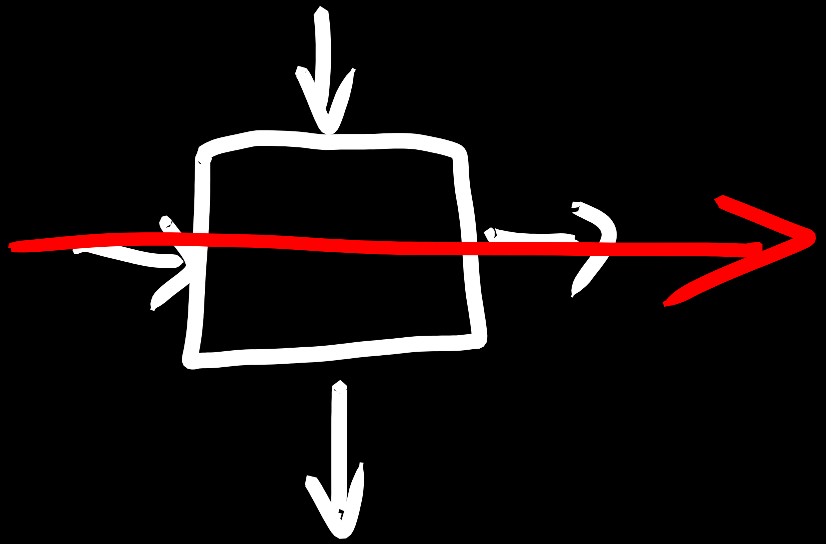
TRICKY

&

HARD

$$U = \sum_i |i\rangle\langle i| \otimes V_i \qquad q_i = \langle i|\xi|i\rangle$$

Then the result of any kind of tomography will be V_i with probability q_i



is contractive

for $\bar{\rho} = \sum_i \rho_i p_i$

then $\bar{\xi}$ is unique

$\Rightarrow$ if $\bar{\rho} \approx \mathbb{1}$

then $\bar{\xi} \approx \mathbb{1}$

$$\mathcal{H}_2 \otimes \mathcal{H}_2$$

can be done analytically

Control-U

contractive

Thanks!