The Fermionic Quantum Theory

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QUit quantum information theory group

- Fermionic systems in computation and physics

- Fermionic Quantum theory
  - Parity SSR in the Wick sense

- Consequences of parity SSR
  - Violation of local tomography
  - Violation of entanglement monogamy

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

The question is, if we wrote a Hamiltonian which involved only these operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I'm not sure whether Fermi particles could be described by such a system. So I leave that open. Well, that's an example of what I meant by a general quantum mechanical simulator. I'm not sure that it's sufficient, because I'm not sure that it takes care of Fermi particles.

Physics	Computation

• Relativity (spin-statistics)
  • Fermionic anticommuting fields
  • Parity superselection rule	

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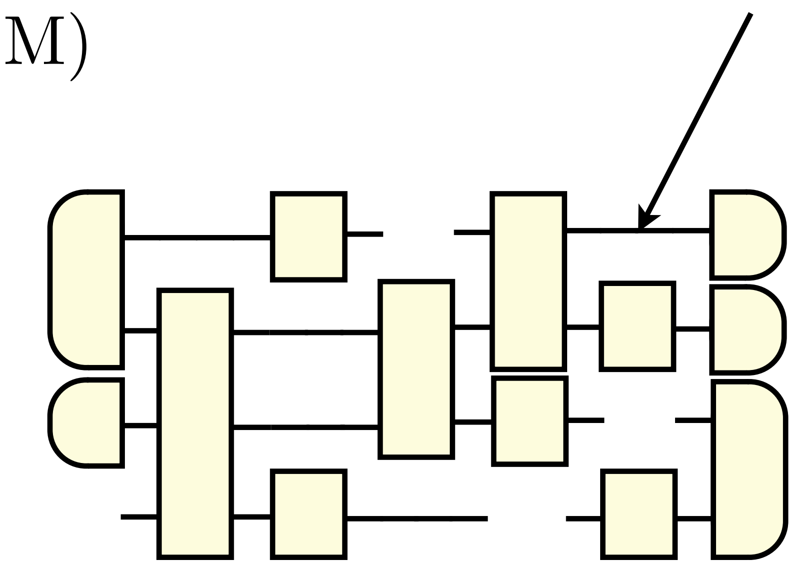
Computation

- Computation: local fermionic modes (LFM)



- Bravyi-Kitaev:
  - Universal fermionic computation
  - Fermionic computation is equivalent to Quantum computation

S. B. Bravyi and A. Y. Kitaev, Annals of Physics 298, 210 (2002)

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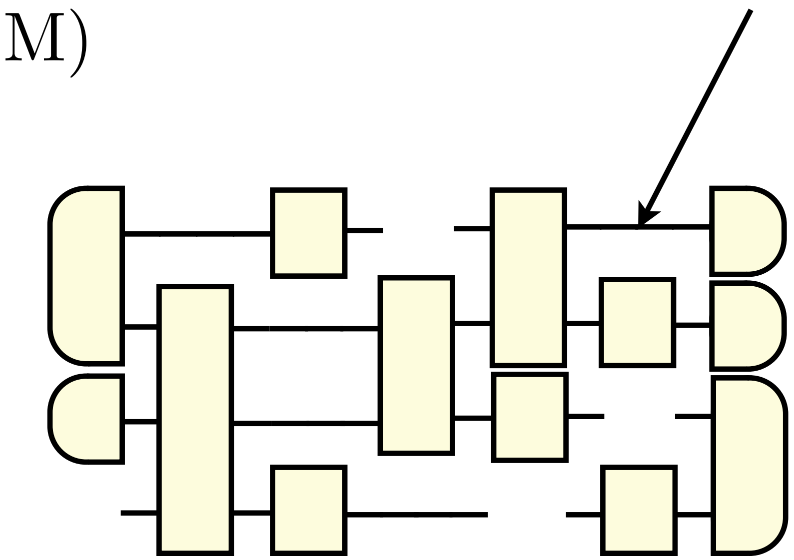

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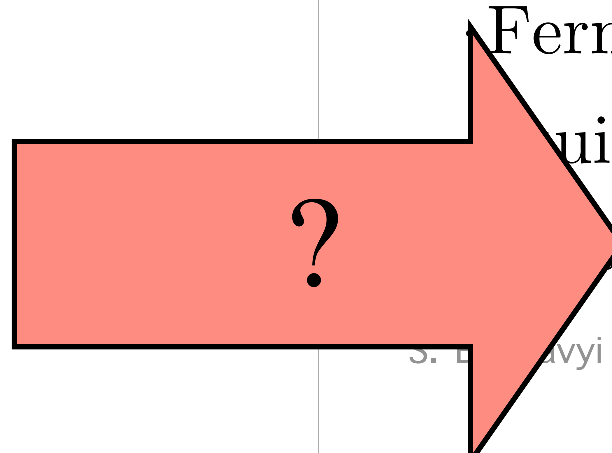

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Jordan-Wigner isomorphism:

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- Fermionic algebra $\{\varphi_i, \varphi_j\} = 0 \quad \{\varphi_i, \varphi_i^\dagger\} = \delta_{ij} I$

- Represent on Fock space $\mathcal{F}_N \sim \mathbb{C}^{2N}$

$$|s_1, \ldots, s_N\rangle := (\varphi_1^\dagger)^{s_1} \cdots (\varphi_N^\dagger)^{s_N} |000 \cdots 0\rangle$$

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Local fermionic operations into nonlocal quantum operations

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Local fermionic operations into nonlocal quantum operations

- On the top impose the Parity SSR

What do we map?

Where does SSR come from?

Fermionic Quantum Theory

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

Fermionic Quantum Theory

- States, effects and maps in terms of φ_i and $\varphi_i^\dagger$

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Fermionic Quantum Theory

- States, effects and maps in terms of φ_i and $\varphi_i^\dagger$

- States of N-LFMs: density matrixes on the Fock space $\mathcal{F}_N \sim \mathbb{C}^{2N}$

$$\rho = \sum_{st} \rho_{st} \prod_{i=1}^{N} \varphi_i^{\dagger\, s_i} \varphi_i \varphi_i^\dagger \varphi_i^{\, t_i} \qquad \rho_{st} \in \mathbb{C}$$

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- $\mathrm{Prob}(\rho, a) := \mathrm{Tr}[a\rho]$

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- $\mathrm{Prob}(\rho, a) := \mathrm{Tr}[a\rho]$

- Transformations $\subset$ linear Hermitian preserving maps: **KRAUS FORM**

$$\mathcal{T}(\rho) = \sum_i s_i K_i \rho K_i^\dagger \qquad s_i = \pm 1$$

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Assumptions on maps

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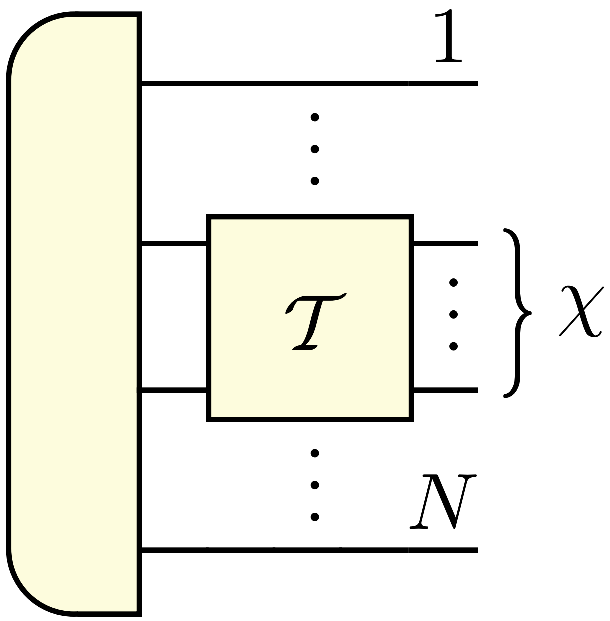
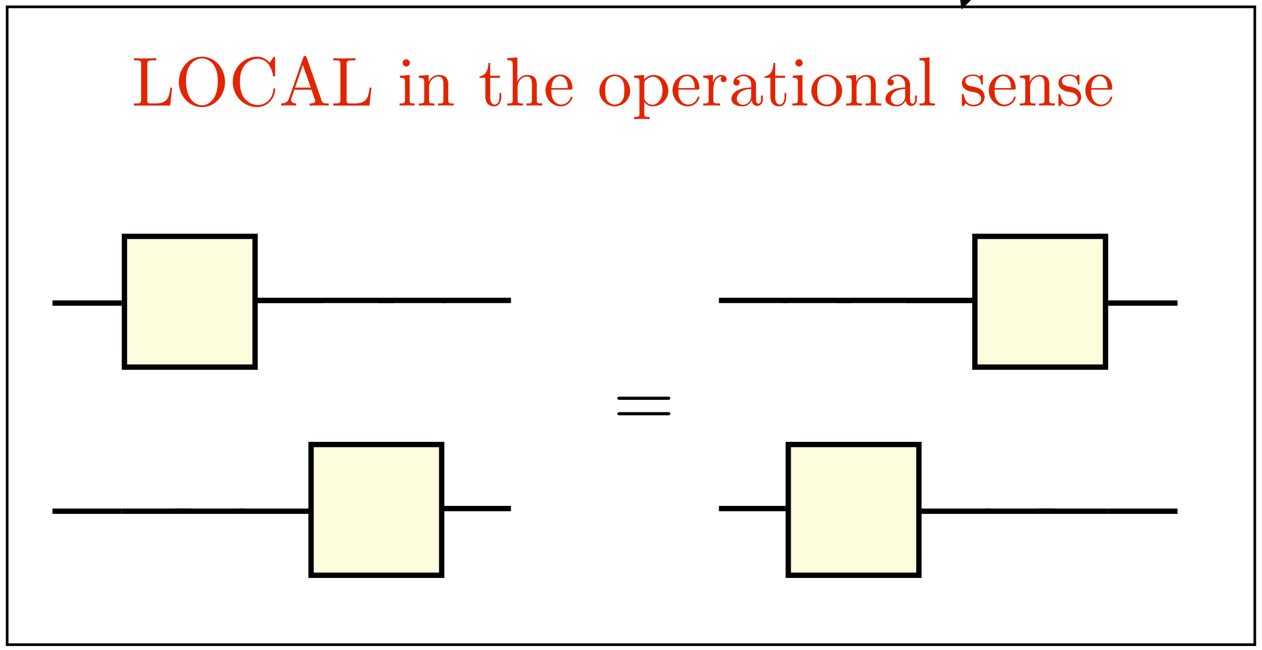
- Maps with Kraus operators involving only $\varphi_i, \varphi_i^\dagger$ with $i \in \chi$ are LOCAL on the LFMs in χ





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Derivation of parity SSR

Proposition. No map can have Kraus that are combination of even and odd products of fields.

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Proof.

$$\mathcal{T}(\rho) = \sum_i s_i K_i \rho K_i$$

if $K_i = E_i + O_i$ for some i

E_i even number of field operators

O_i odd number of field operators

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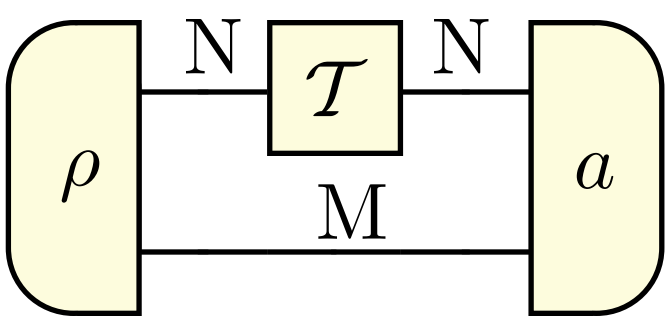
then 

$= 0 \qquad \forall \rho \in \mathrm{St}(\mathrm{NM}), \quad \forall a \in \mathrm{Eff}(\mathrm{NM})$

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Derivation of parity SSR

Corollary. States must be combination of even products of field operators.

$$\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$$

$$\rho = \left(\begin{array}{c|c} p\rho_e & 0 \\ \hline 0 & (1-p)\rho_o \end{array}\right) \qquad \rho = p_e\rho_e + p_o\rho_o \qquad p_o + p_e = 1$$

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Set of States of N-LFMs $\quad \mathrm{St}_{\mathbb{R}}(\mathrm{N_F}) = \mathrm{Herm}(\mathbb{C}^{2^{N-1}}) \oplus \mathrm{Herm}(\mathbb{C}^{2^{N-1}})$

$\cong$ N-1 qubit state spaces

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Parity SSRs is not conservation of Parity

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1-LFM
=
calssical bit

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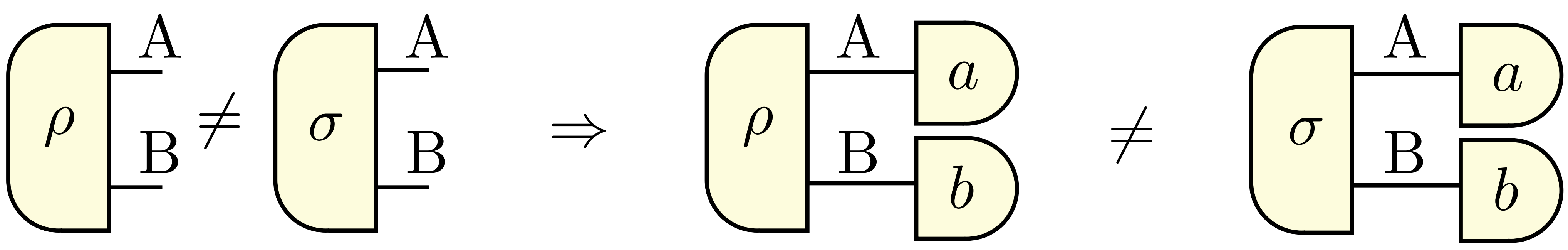
Fermions violate local tomography

L. Hardy and W. K. Wootters, Foundations of Physics 42, 454 (2012)

Fermions violate local tomography

- Local tomography:



$$D_{\mathrm{AB}} = D_{\mathrm{A}} D_{\mathrm{B}}$$

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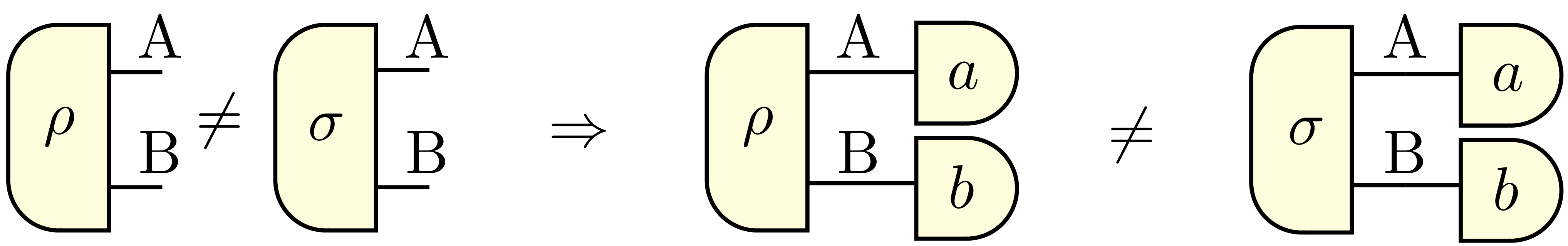

$$D_{\mathrm{AB}} = D_{\mathrm{A}} D_{\mathrm{B}}$$

- Bilocal tomography: I need local and bilocal effects for state tomography

$$D_{\mathrm{AB}} > D_{\mathrm{A}} D_{\mathrm{B}}$$

$$D_{\mathrm{ABC}} \leqslant f(D_{\mathrm{A}}, D_{\mathrm{B}}, D_{\mathrm{C}}, D_{\mathrm{AB}}, D_{\mathrm{AC}}, D_{\mathrm{BC}})$$

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Quantum theory

N-qubits

$$\mathrm{St}_{\mathbb{R}}(\mathrm{N_Q}) = \mathrm{Herm}(\mathbb{C}^{2^N})$$

$$D_{\mathrm{N_Q}} = 2^{2N}$$

local tomographic

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Fermionic Quantum theory

N-LFMs

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$$D_{\mathrm{N_F}} = 2^{2N-1} = \frac{1}{2} D_{\mathrm{N_Q}}$$

bilocal tomographic

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Fermionic entanglement

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Fermionic entanglement

Oper. prob. theory: 1) provide a notion of entanglement

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1)**Proposition.** Non-separability as the unique notion of fermionic entanglement

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1)**Proposition.** Non-separability as the unique notion of fermionic entanglement

2)**Entanglement cost:** resource under LOCC operations

$$|\Psi\rangle_{res}{}^{\otimes N} \xrightarrow{\text{LOCC}} \rho^{\otimes M}$$

N of "resource states" (ebits)

M copies of ρ

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Quantum entanglement of formation

W. K. Wootters, Quantum Information & Computation 1, 27 (2001)
S. Popescu and D. Rohrlich, Phys. Rev. A 56, R3319 (1997)

Quantum entanglement of formation

Asymptotical operational meaning:

$$|\Psi\rangle_{res}^{\otimes N} \xrightarrow{\text{LOCC } \mathcal{D}_\rho} \rho^{\otimes M} \qquad E(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

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$$E(|\Psi\rangle) = S(\mathrm{Tr}_A |\Psi\rangle\langle\Psi|)$$ pure states

$$E(\rho) := \min_{\mathcal{D}_\rho} \sum_i p_i E(|\Psi_i\rangle)$$ mixed states

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$$E(\rho) = 0 \Leftrightarrow \rho \text{ separable}$$
$$E(\rho) = 1 \Leftrightarrow \rho \text{ maximally entangled}$$

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Quantum: 2-qubits

Maximally entangled state

unique bipartite state
LOCC convertible to any other

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

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Fermionic: 2-LFMs

Maximally entangled sets*

not unique bipartite state
LOCC convertible to any other

$$\{\alpha\,|00\rangle + \beta\,|11\rangle\,, \alpha, \beta > 0\}$$

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Fermionic entanglement of formation

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M.-C. Bañuls, J. I. Cirac and M. M. Wolf, Phys. Rev. A 76, 022311 (Aug 2007)

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$$E_{\mathrm{F}}(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

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$$|\Psi\rangle_{res}^{\otimes N} \xrightarrow{\text{LOCC}_\text{F} \quad \mathcal{D}_\rho^\text{F}} \rho^{\otimes M} \qquad E_\text{F}(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

Proposition. $\tilde{E}_\text{F}(\rho) \leqslant E_\text{F}(\rho)$

$$\tilde{E}_\text{F}(\rho) = p_e E(\rho_e) + p_o E(\rho_o)$$

$$\rho \rightarrow p_e \rho_e + p_o \rho_o$$

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)
M.-C. Bañuls, J. I. Cirac and M. M. Wolf, Phys. Rev. A 76, 022311 (Aug 2007)

Maximally entangled mixed state

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, arXiv preprint arXiv:1307.7902 (2013)

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$$\rho_\star = \frac{1}{2}\rho_e + \frac{1}{2}\rho_o$$

$$|\Psi_e\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |\Psi_o\rangle \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

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Fermionic entanglement is not monogamous

V. Coffman, J. Kundu and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)

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  3-qubits:

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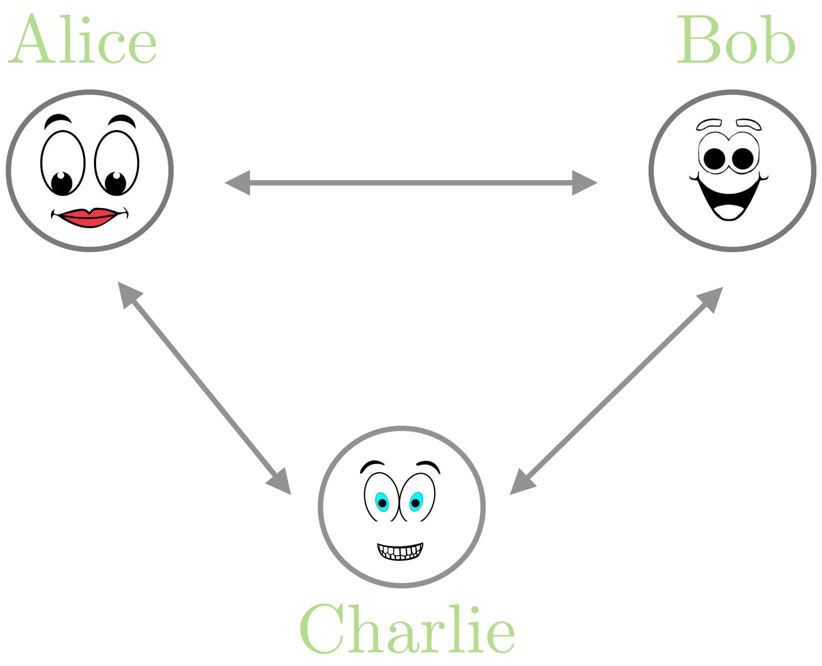

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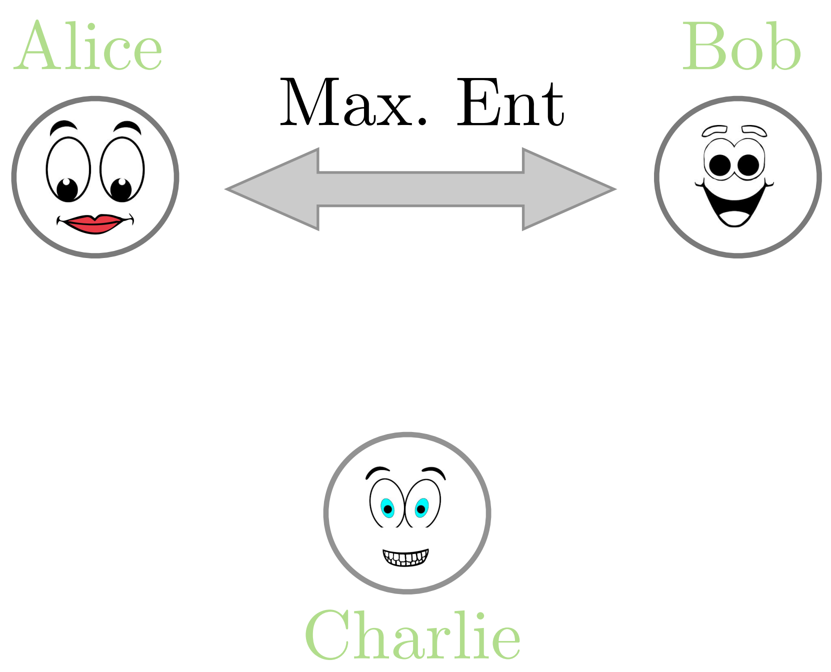

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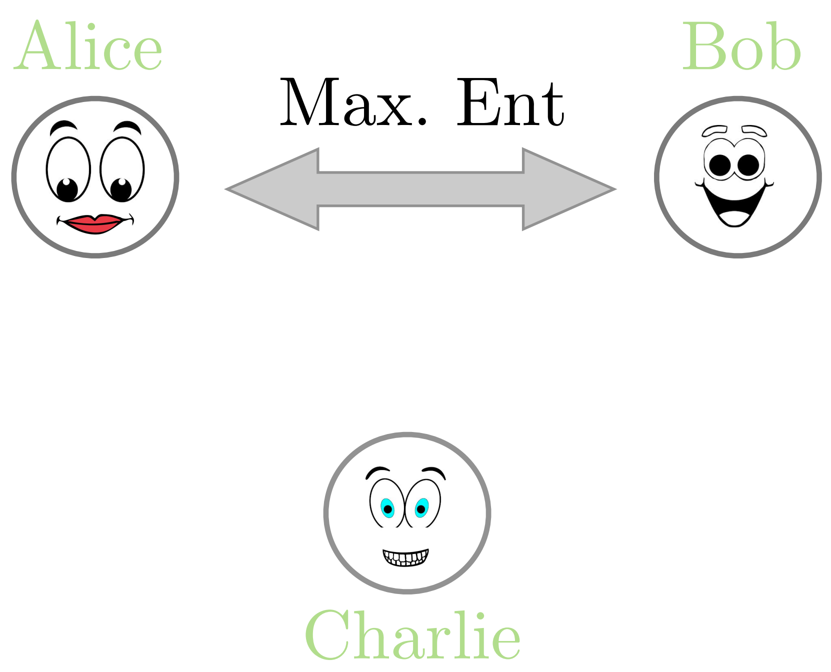

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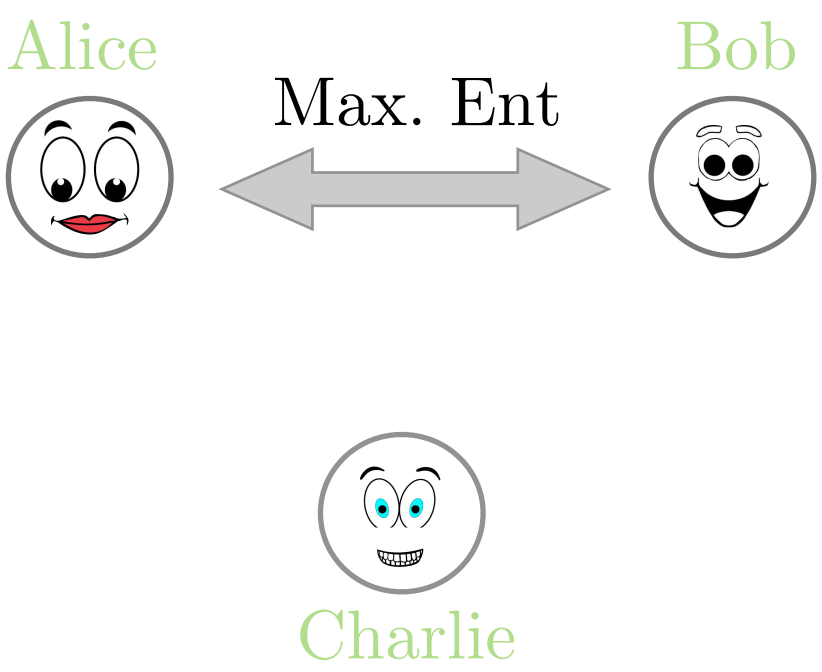

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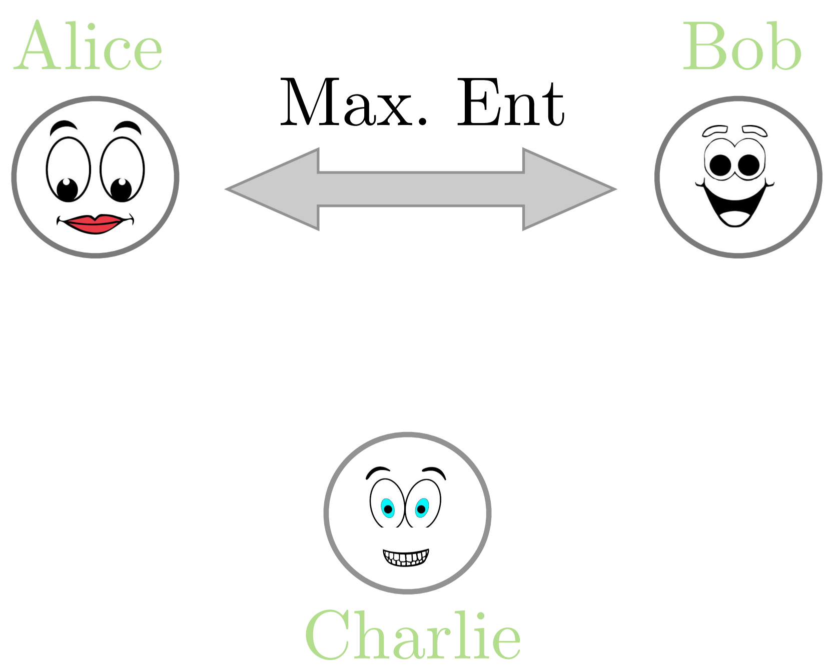

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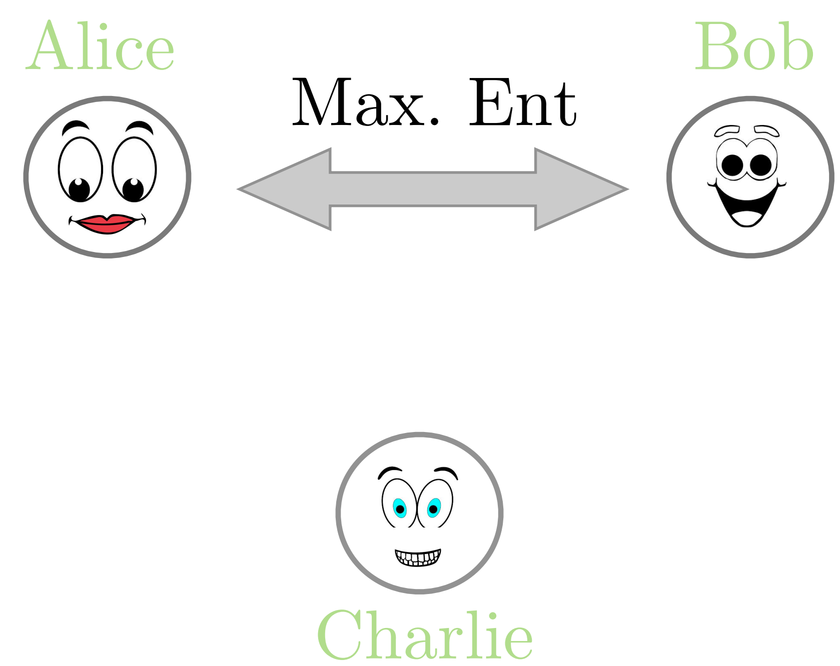

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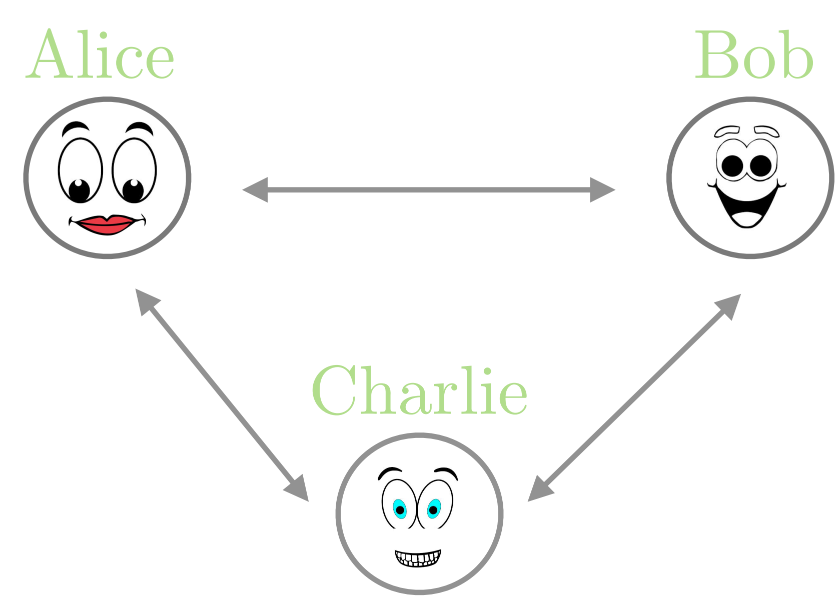
3-LFMs:

$$|\Psi\rangle_{\mathrm{ABC}} = \tfrac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$$



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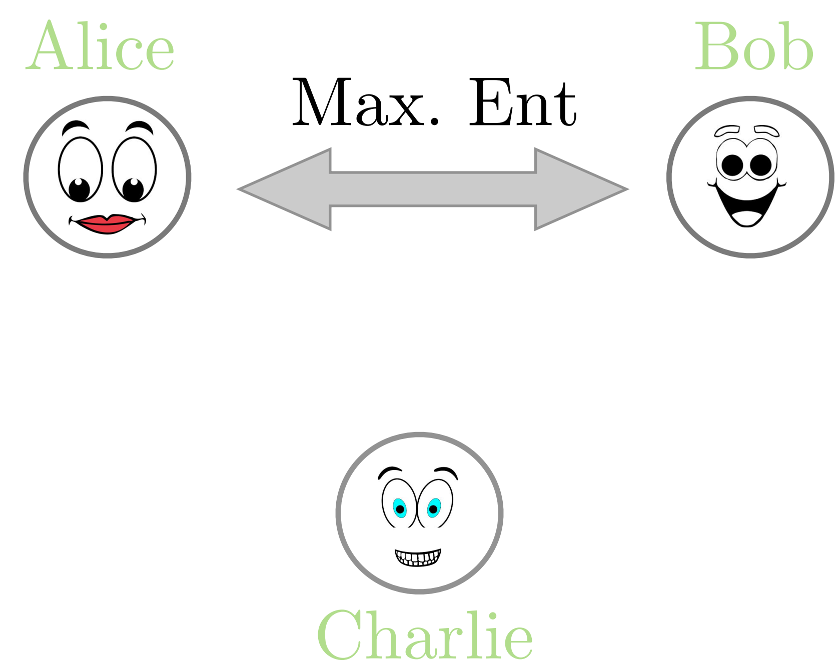

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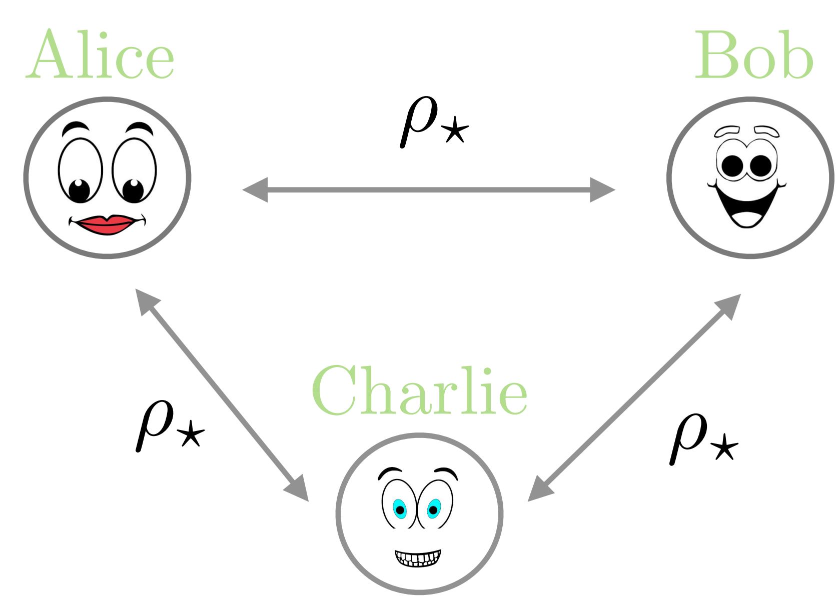

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Conclusions

- Wick parity SSR for fermionic systems derived in an operational context

- Fermionic Quantum theory differs from Quantum theory:
  - it does not satisfy local tomography
  - fermionic entanglement is not monogamous

- Possible applications of these two features?
  - Computation: model alternative to the one based on qubits,
    e.g. cryptography in a Fermionic scenario.
  - Physics: black hole information

P. Hayden and J. Preskill, J. High Energy Phys. 09 (2007) 120
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Thanks!