Locking vs. private capacity of quantum channels

Andreas Winter (ICREA & UAB Barcelona)

[Based on S. Guha et al., arXiv:1307.5368
and AW, arXiv:1403.6361]

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Poster by Cosmo Lupo

Outline

1. Quantum channels and capacities

2. Notions of privacy

3. The (weak) locking capacity

4. An achievable rate for L_W: two examples

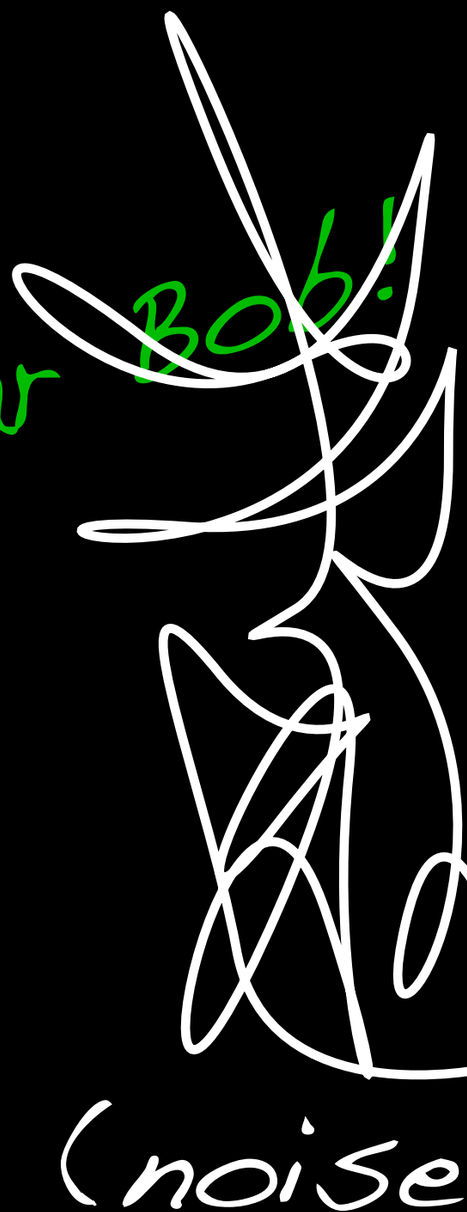
5. Discussion

1. Communication

Dear Bob!

Shannon (1948): Fundamental problem is that of reproducing at one point a message selected at another point.

1. Communication

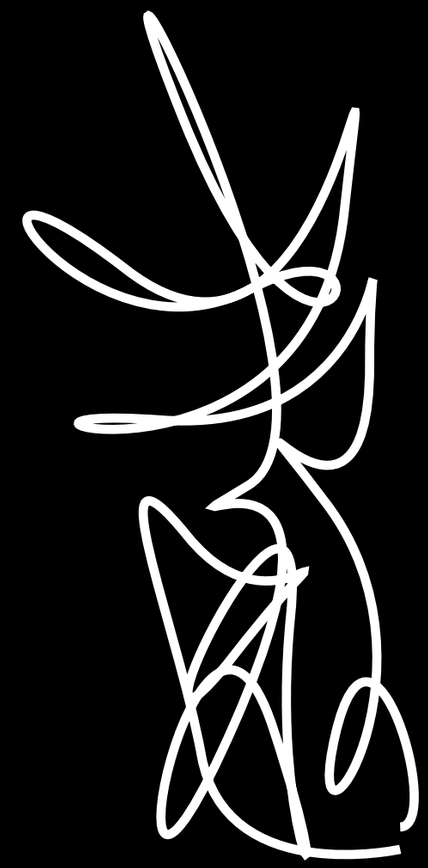


1. Communication

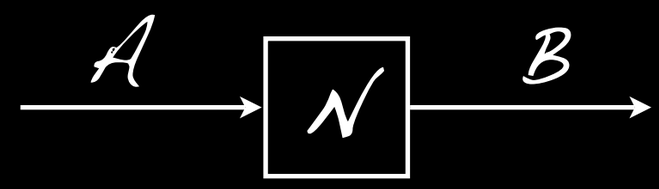
Deep throat?

1½. Channels & capacities

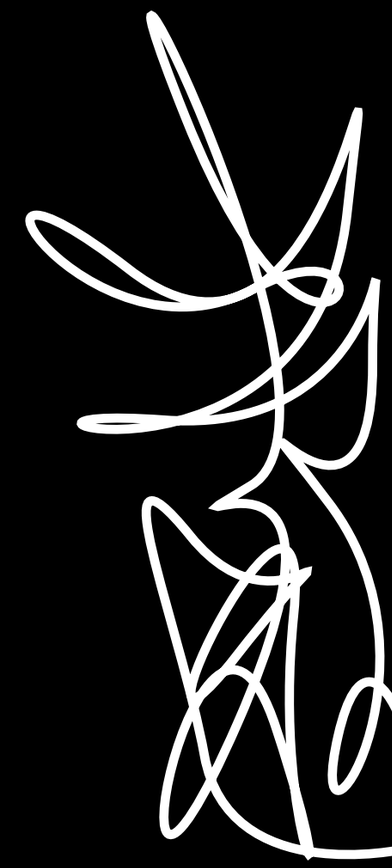
Noise

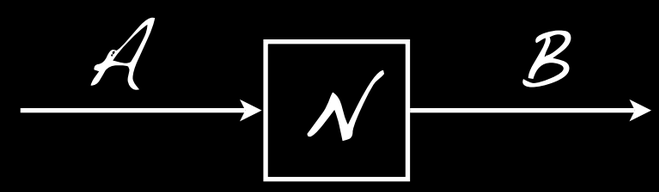


modelled as "channel":



1½. Channels & capacities

Noise  modelled as "channel":



Channel = cptp map $N: L(A) \longrightarrow L(B)$,

where A, B are finite-dim. Hilbert spaces.

Kraus representation, you know...

1½. Channels & capacities

Stinespring: $N(\rho) = Tr_E\, V \rho V^\dagger$,

with an isometry $V : A \hookrightarrow B \otimes E$.

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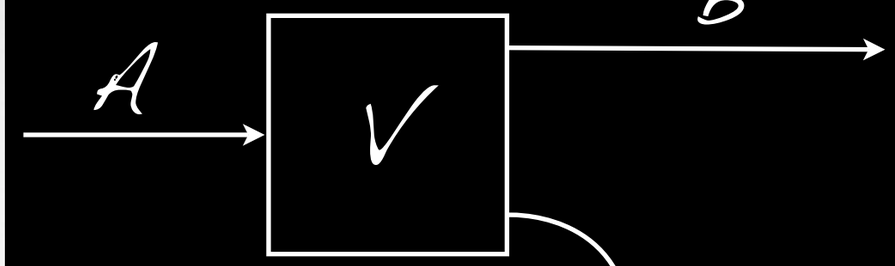

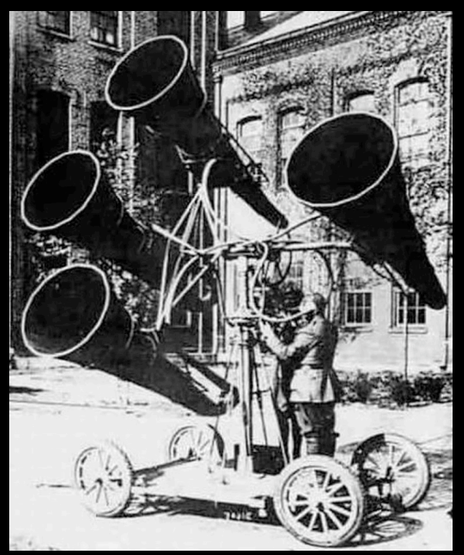

1½. Channels & capacities

Ex: 1) Noiseless channel = identity id_A.

2) Constant channel $K(\rho) = \omega_0$.

3) Depolarizing channels

4) Amplitude damping channels

5) Phase damping channels

6) Erasure channel $\epsilon_g(\rho) = (1-g)\rho \oplus g|*\rangle\langle *|$

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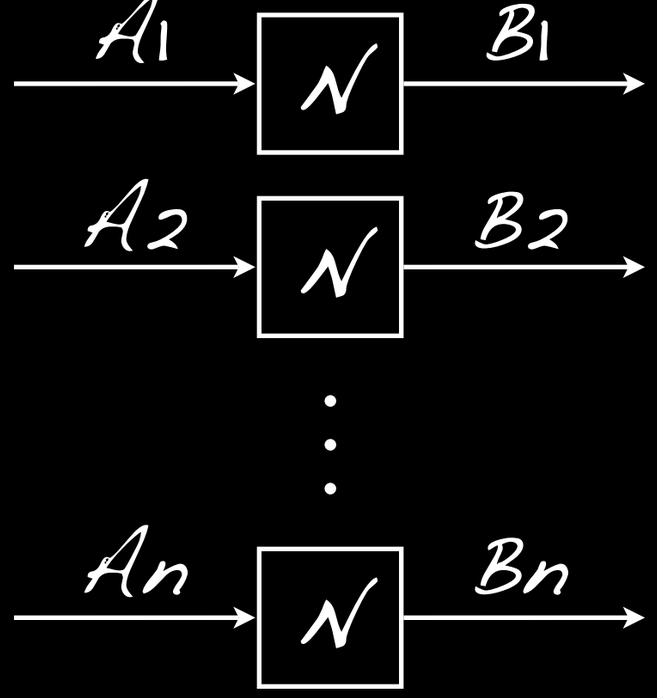

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message of $k=k(n,\varepsilon)$ bits

$=m$ with prob $\geq 1-\varepsilon$

Thm (Holevo and Schumacher/Westmoreland, 1973 and 1996/7):

$$C(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n}\chi(\mathcal{N}^{\otimes n}), \text{ with}$$

$\chi(\mathcal{N}) = \max I(X:B)$ wrt. $\{p_x, \rho_x\}$ and

$$\rho_{XB} = \sum_x p_x |x\rangle\langle x| \otimes \mathcal{N}(\rho_x).$$

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Holevo information $S(\rho^B) - \sum_x p_x S(\mathcal{N}(\rho_x))$

Von Neumann entropy: $S(\rho) = -\mathrm{Tr}\ \rho \log \rho$

Unfortunately,

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However, for some classes of channels it is, and we know the classical capacity $C(\mathcal{N})$ as $\chi(\mathcal{N})$.

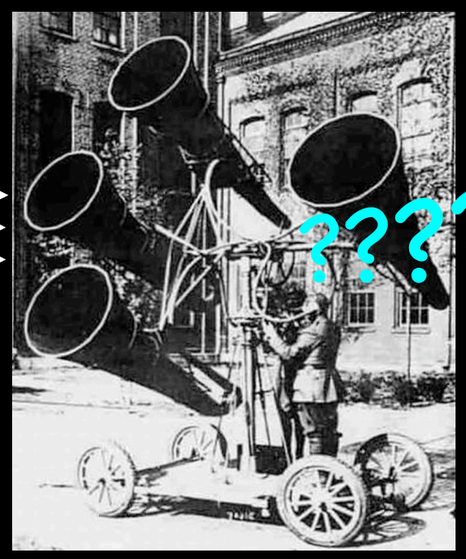
Private capacity $P(N)$:= maximum cbit rate $\frac{k}{n}$ for asymptotically error-free and secret transmission over $N^{\otimes n}$.

m → ρ_m

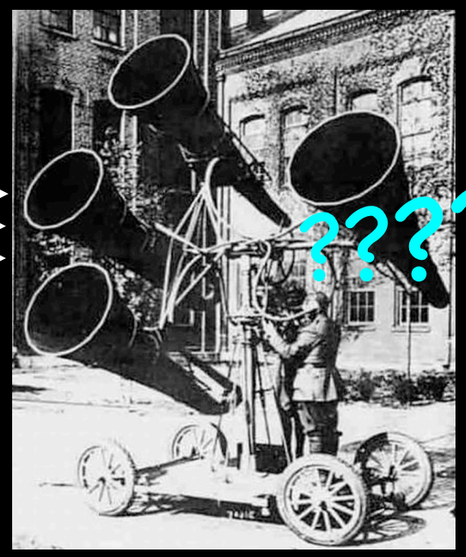
$\hat{m}$

error $\leq \varepsilon$

message of $k=K(n,\varepsilon)$ bits



??????



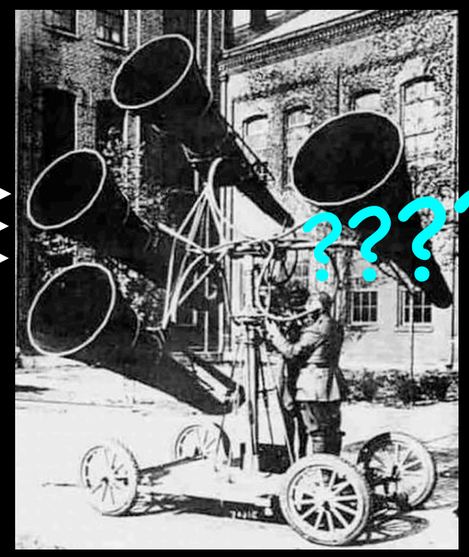
$m \rightarrow \rho_m$

A_1 A_2 A_n — V — B_1 B_2 B_n E_1 E_n

$D \rightarrow \hat{m}$

??????

In QKD: Eve provides the channel; i.i.d. means (known) "collective attack".



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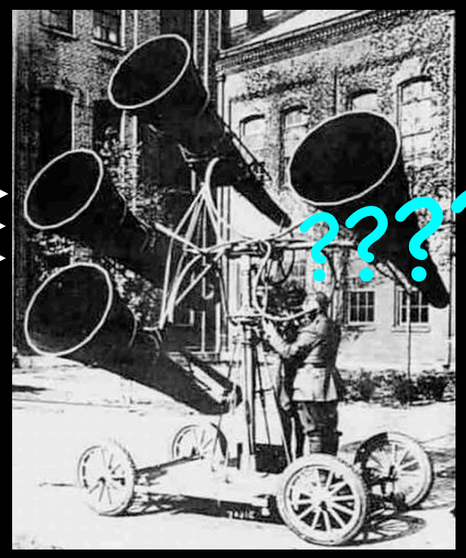
Privacy definition is subtle...

$m \longrightarrow \rho_m$ $\quad A_1 \to V \to B_1$ $\quad A_2 \to V \to B_2$ $\quad \vdots$ $\quad A_n \to V \to B_n$ $\quad D \to \hat{m}$

$V \to E_1, \ldots, E_n$



??????

Privacy definition: In the past, would assume that "at the end", Eve measures, so demand $I_{acc}(M:E^n) \approx 0$.

Nowadays: $I(M:E^n) \approx 0$.

2. Notions of privacy

Assume that messages $m=1,\dots,M$ are uniformly distributed (as if it was secret key). Then, at the end of any protocol, Alice, Bob & Eve share a state

$$\omega = \frac{1}{M}\sum_m |m\rangle\langle m|^{AB} \otimes \rho_m^E$$

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Neglect errors between A & B ✓

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Old style: Eve has to measure, some POVM (Q_j); require that j almost independent of m.

$$\omega = \frac{1}{M} \sum_m |m\rangle\langle m|^{AB} \otimes \rho_m^E$$

$$\Pr\{M=m,\ J=j\} = \frac{1}{M} \mathrm{Tr}\ \rho_m Q_j$$

Require: $\max_Q I(M:J) =: I_{acc}(M:E) \leq \varepsilon \quad (*)$

[Cf. Mayers, J. ACM 2001;
Scarani et al. Rev. Mod. Phys, 2009]

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Information locking [DiVincenzo et al., PRL 2004; Hayden et al., CMP 2004; Koenig et al., PRL 2007]: states uniformly mixed over k d-dimensional bases, $k<\mathrm{polylog}(d)$, with $(*)$.

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Information locking [DiVincenzo et al., PRL 2004; Hayden et al., CMP 2004; Koenig et al., PRL 2007]: states uniformly mixed over k d-dimensional bases, $k<\mathrm{polylog}(d)$, with $(*)$. Clearly insecure: if Eve can delay measurement until she learns basis, she can measure m!

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Prompted "composable" security definition

[Renner, PhD thesis, quant-ph/0512258]: $I(M:E) \leq \varepsilon$.

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$$I(M:E) = S(\omega^E) - \sum_m p(m)\, S(\rho_m)$$

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This doesn't have the unlocking problem; in fact, says that ρ_m^E almost independent of m.

[More history in review by Scarani et al. Rev. Mod. Phys, 2009]

$$\omega = \frac{1}{M}\sum_m |m\rangle\langle m|^{AB} \otimes \rho_m^E$$

$$Pr\{M=m,\ J=j\} = \frac{1}{M}\,Tr\,\rho_m Q_j$$

Remark: The gap between $I(M:E)$ and $I_{acc}(M:E) = \max_Q I(M:J)$ is precisely the discord of the state ω.

Private capacity $P(N)$:= maximum cbit rate $\frac{k}{n}$ for asymptotically error-free and secret transmission over $N^{\otimes n}$.

Privacy (composable):

$I(M:E^n) \leq \delta$

Capacity formula for $P(N)$:

Thm (Devetak and Cai/Yeung/AW, 2003):

$$P(N) = \lim_{n\to\infty} \frac{1}{n} P^{(1)}(N^{\otimes n}), \text{ with}$$

$$P^{(1)}(N) = \max I(X:B) - I(X:E) \text{ wrt. } \{p_x, \rho_x\}$$

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Quantum analogue of a famous result by Wyner [Bell Syst. Tech. J., 1975] & Csiszár/Körner [IEEE-IT, 1978]. Only that for classical channels, $P^{(1)}$ equals the private capacity; in general, the above has an additivity issue. Also P is not additive!

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For degradable channels (i.e. $\hat{\mathcal{N}} = \mathcal{D}\circ\mathcal{N}$):
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For symmetric $\mathcal{N} = \hat{\mathcal{N}}$: $P(\mathcal{N}) = 0$.

3. (Weak) locking capacity

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Let's take that seriously: "quantum enigma machines" [Lloyd, arXiv:1307.0380; Guha et al., arXiv:1307.5368] ...

Locking capacity $L_W(N)$:= maximum cbit rate $\frac{k}{n}$ for asymptotically error-free and accessible-secret transmission over $N^{\otimes n}$?

m

ρ_m

$k = K(n, \varepsilon)$ bits

A_1 A_2 A_n

V V V

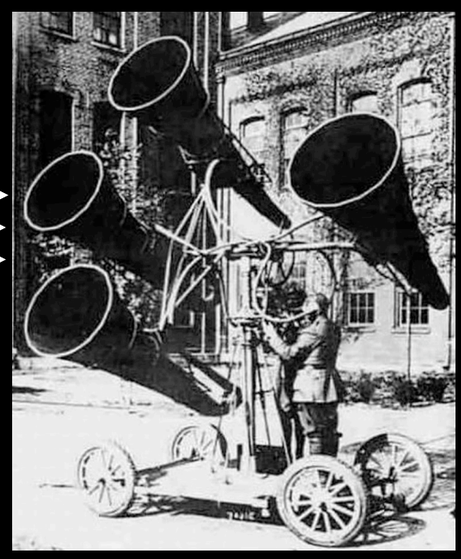
B_1 B_2 B_n

D

$\hat{m}$

error $\leq \varepsilon$

E_1 E_n



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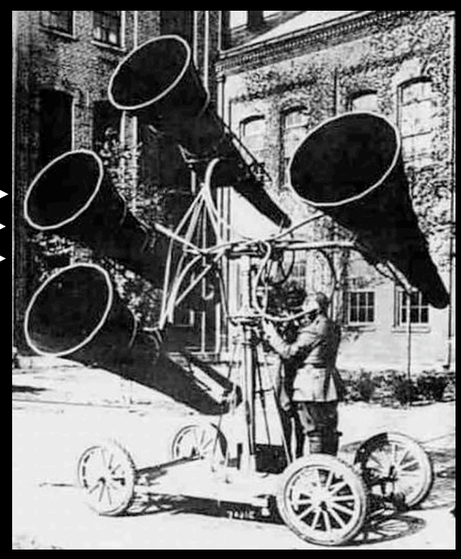

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A&B preshare $o(n)$ key

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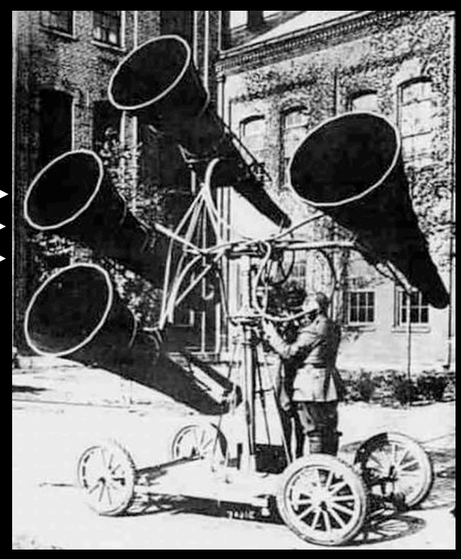

Bounds on $L_w(N)$ - clearly $\geq P(N)$:

* If N is entanglement-breaking, then
  $P(N) = L_w(N) = 0$.

[Guha et al., arXiv:1307.5368]

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* If N is entanglement-breaking, then $P(N) = L_W(N) = 0$.

  [Because then N has rank-one Kraus operators, and so Eve can make a measurement, only depending on N, s.t. Alice & Bob are independent conditioned on Eve's outcome.]

[Guha et al., arXiv:1307.5368]

Bounds on $L_w(N)$ - clearly $\geq P(N)$:

* If N is entanglement-breaking, then $P(N) = L_w(N) = 0$.

* $L_w(N) \leq \lim_{n\to\infty} \frac{1}{n} L_w^{(u)}(N^{\otimes n})$, with

$$L_w^{(u)}(N) = \max I(X:B) - I_{acc}(X:E)$$
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Cf. Csiszár-Körner-Devetak expression

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Not known to be attainable! Before, no separation between P and L_W was known.

[Guha et al., arXiv:1307.5368]

4. Achievable rates for L_W

Summary of main findings:

1) A family of channels with $P(N) = 1$, but $L_W(N) = 1 + \frac{1}{2} \log d$.

2) A symmetric channel, hence $P(N) = 0$, but $L_W(N) > 0$.

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Complementary $\hat{\mathcal{N}}$ is cq-channel:

$\hat{\mathcal{N}}(|i\rangle\langle j|) = \delta_{ij}\,|\psi_i\rangle\langle\psi_j|$

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Degradable, thus $P(\mathcal{N})$ easy to compute.

Use only $|i\rangle\langle i|$ signals: no noise for Bob;

privacy from non-orthogonality of $|\psi_i\rangle$...

1) A family of channels with $P(\mathcal{N}) = 1$, but

$$L_W(\mathcal{N}) = 1 + \tfrac{1}{2}\log d.$$

$$\mathcal{N}(|i\rangle\langle j|) = \langle\psi_j|\psi_i\rangle\,|i\rangle\langle j|.$$

$$\widehat{\mathcal{N}}(|i\rangle\langle j|) = \delta_{ij}\,|\psi_i\rangle\langle\psi_j|$$

Alice: $I \sim (p_i)$

Average output:

$$\rho = \sum_i p_i \psi_i$$

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$$L_W(\mathcal{N}) = 1 + \tfrac{1}{2}\log d.$$

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[Damgaard et al., arXiv:quant-ph/0612014]

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of Guha et al.: $L_W(N) = 1 + \frac{1}{2} \log d$.

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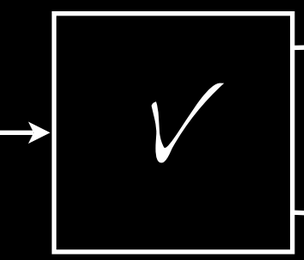

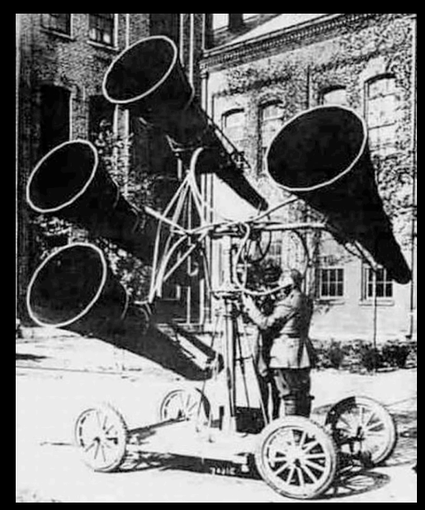

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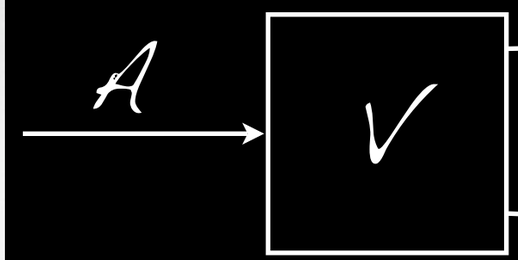

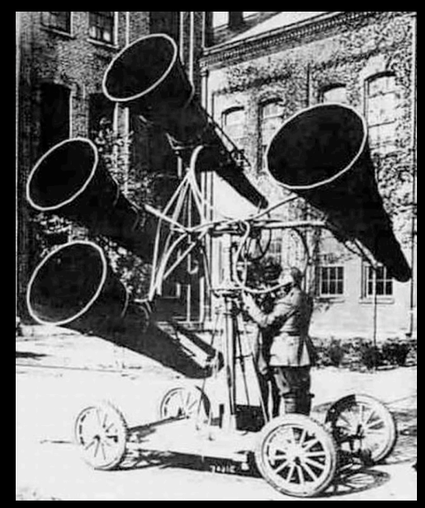

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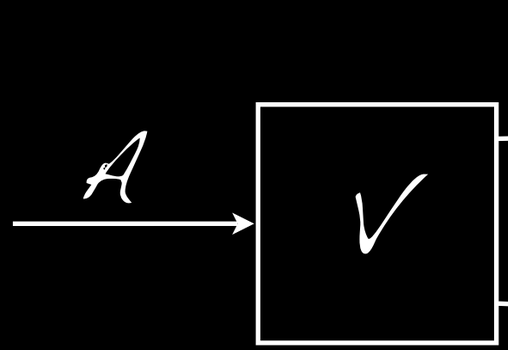

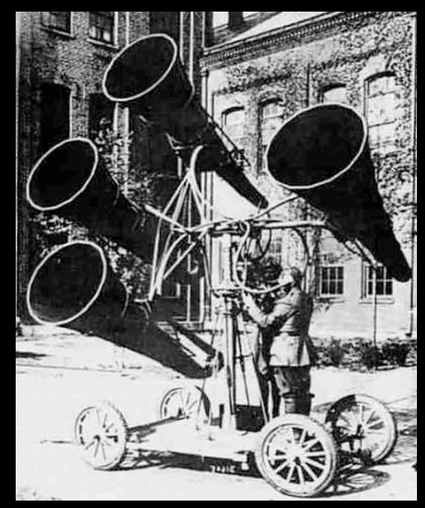

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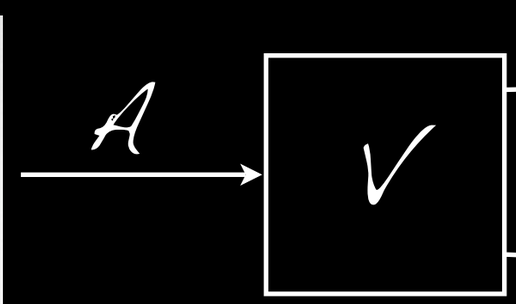

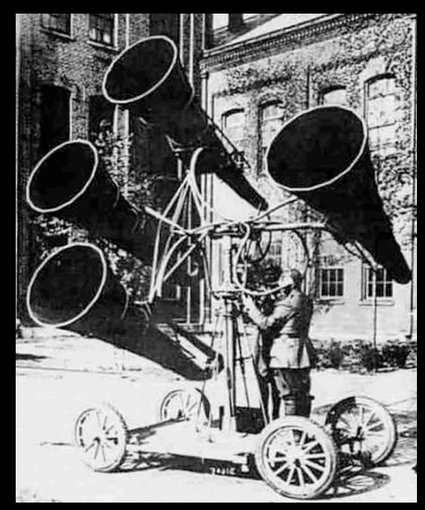

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- Compare with Koenig et al. [PRL, 2007], where it was shown in principle that locking can make a key appear secure. Here: realistic scheme of i.i.d. channels (i.e. collective attack).

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- Conjecture that Guha et al.'s upper bound

$$L_{w}^{(u)}(\mathcal{N}) = \max I(X:B) - I_{acc}(X:E)$$
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- Conjecture: for any i.i.d. ensemble $\{p_x, \rho_x\}^{\otimes n}$ and POVM (Q_j) on E^n,

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- ...would follow from additivity of minimum conditional Rényi entropy $\min H_\alpha(X|J)$ wrt (Q_j) on E, $\alpha > 1$.

[Cf. AW, arXiv:1403.6361 & King, QIC 2003]