Self-correcting quantum memories in 3 dimensions or (slightly) less

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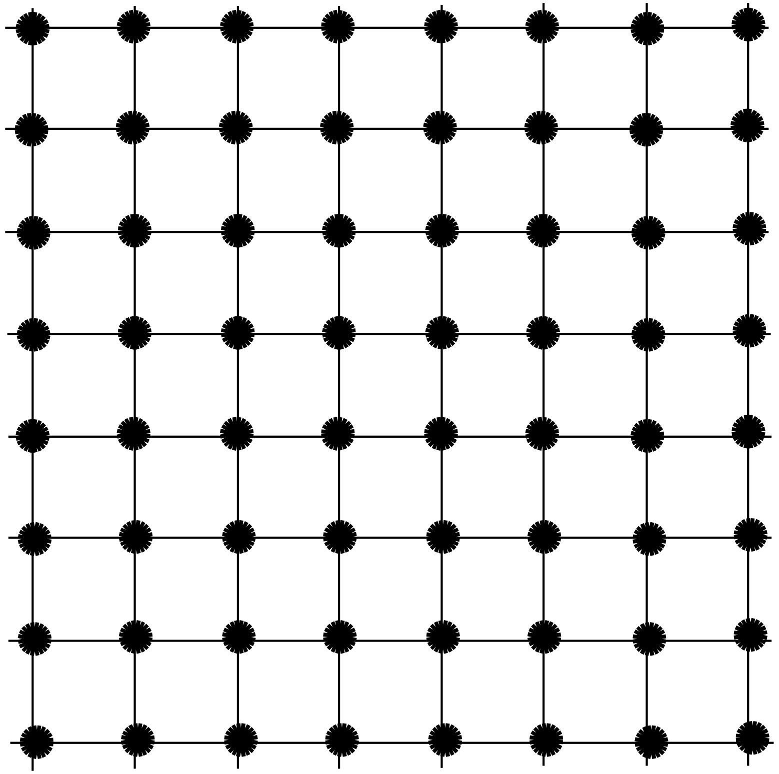
arXiv: 1411.7046

Self-correcting classical memories



2D Ising model

$$H = -\sum_{i\sim j} Z_i Z_j$$

- 2-fold degenerate
- Extensive distance
- Finite temperature ordered phase
- Exponential memory lifetime

} Zero temperature
.

} Finite temperature

Self-correcting quantum memories

4D toric code (Dennis et al. quant-ph/0110143)

Qubits on faces

X-stabilizers on links $A_l = \prod_{j \in l} X_j$

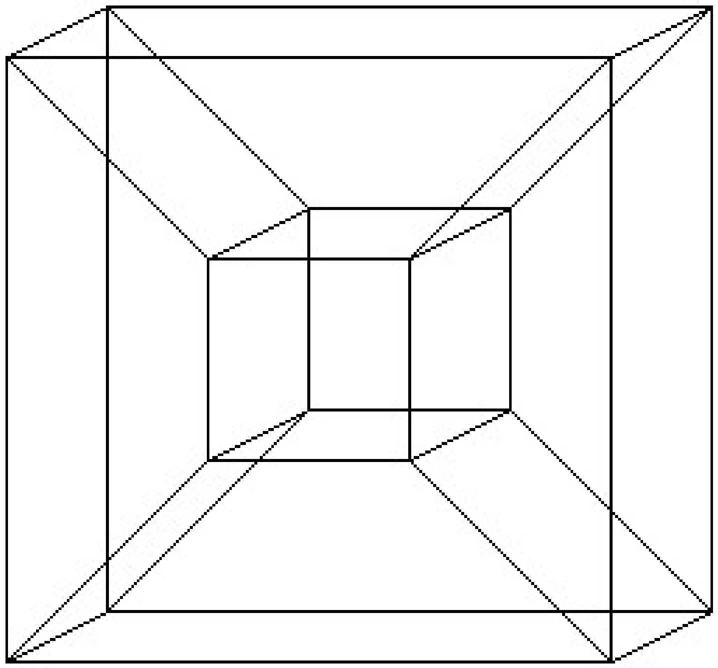
Z-stabilizers on cubes $B_c = \prod_{j \ni c} Z_j$

$$[A_l, B_c] = 0$$

- $$H = -\sum_l A_l - \sum_c B_c$$

2 phase transitions

Exponential lifetime (Alicki et al. 0811.0033)



But what about 3D?

The Caltech Rules

1. Finite spins
2. Bounded local interactions
3. Nontrivial codespace
4. Perturbative stability
5. Efficient decoding
6. Exponential lifetime

phase transitions

Quantum is the square of classical

Quantum is the square of classical

4D toric code   2D Ising model

Quantum is the square of classical

4D toric code 2D Ising model

2D toric code 1D Ising model

Quantum is the square of classical

4D toric code — 2D Ising model

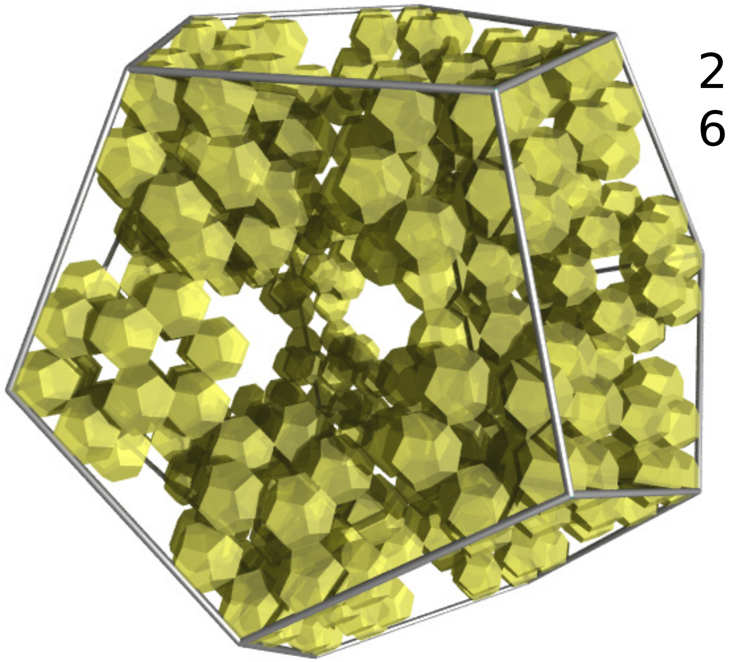
3D quantum memory

2D toric code — 1D Ising model

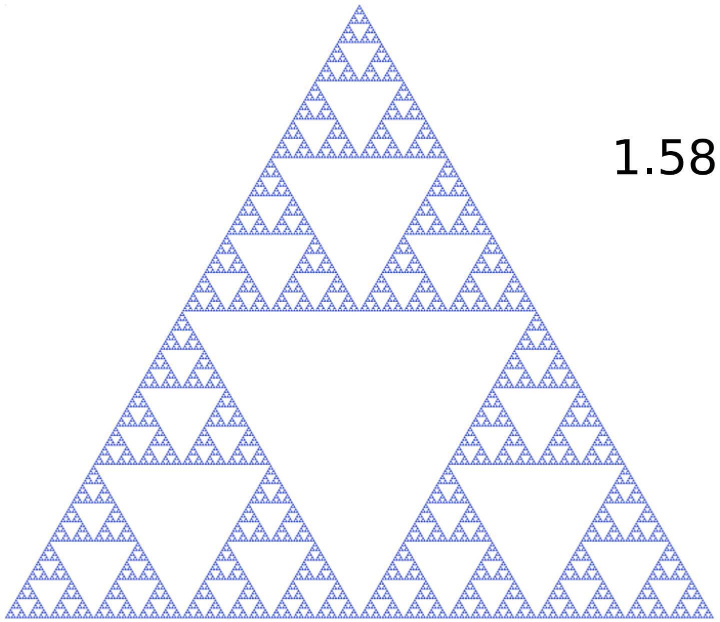
Quantum is the square of classical

4D toric code		2D Ising model
3D quantum memory	?	1.5D classical memory
2D toric code		1D Ising model

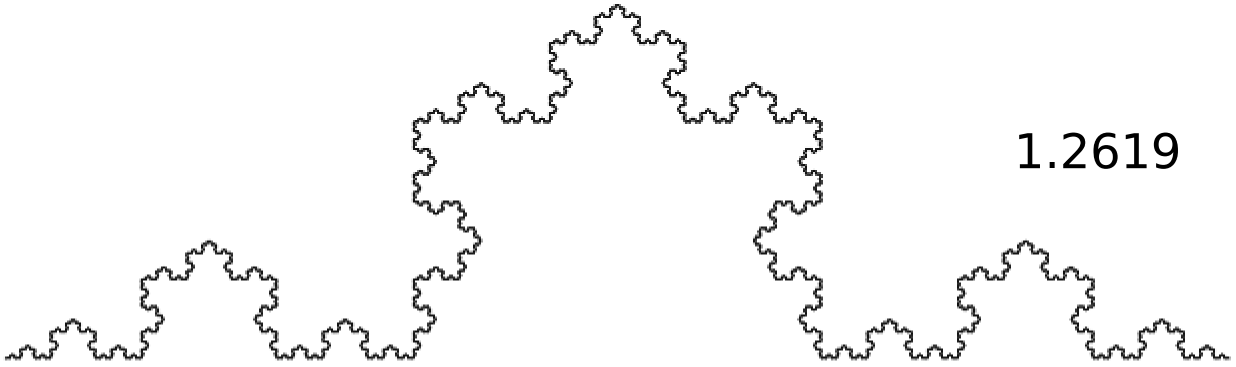
Fractal geometry



2.329
6



1.5849



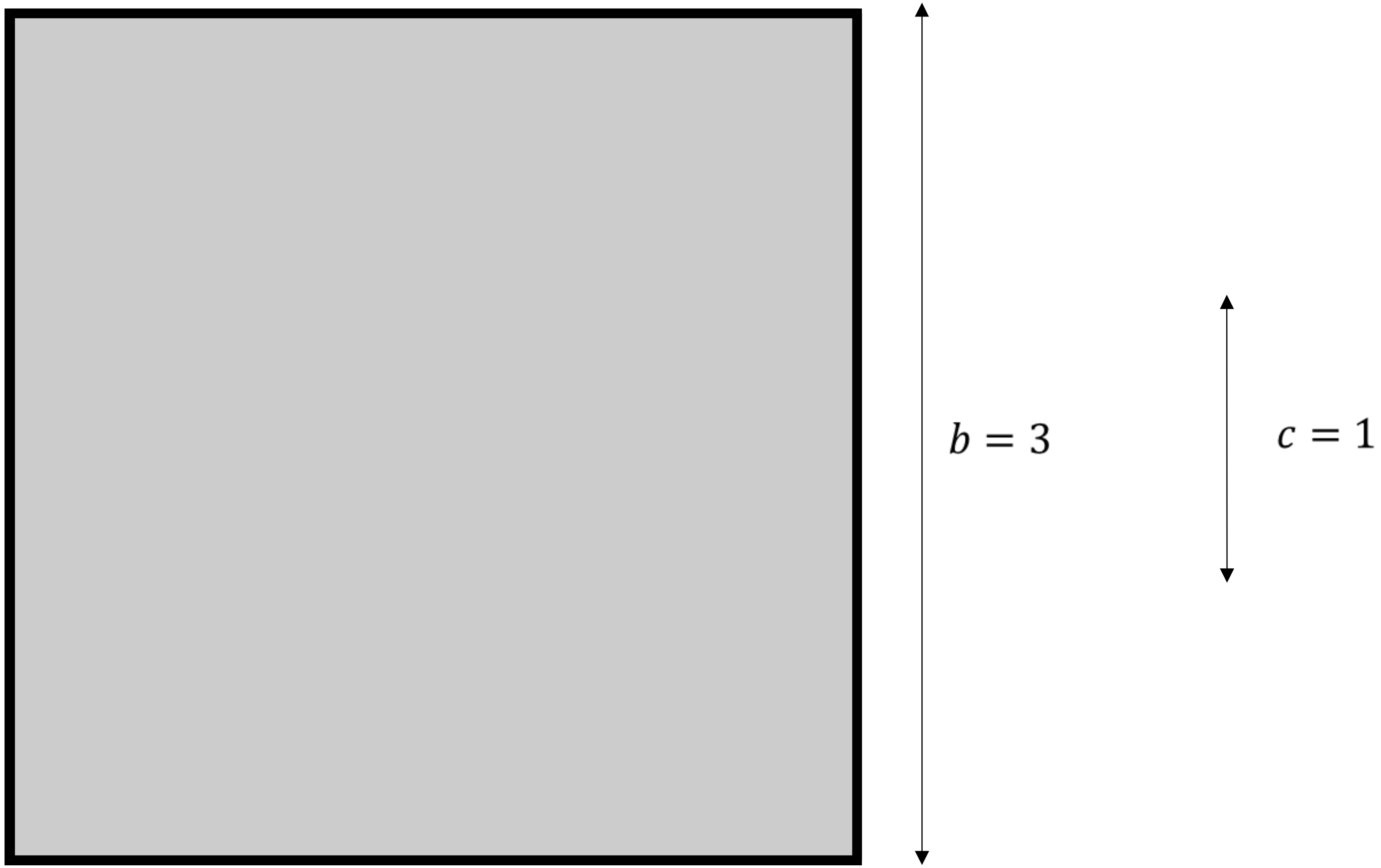
1.2619

Hausdorff dimension

Sierpinski carpet

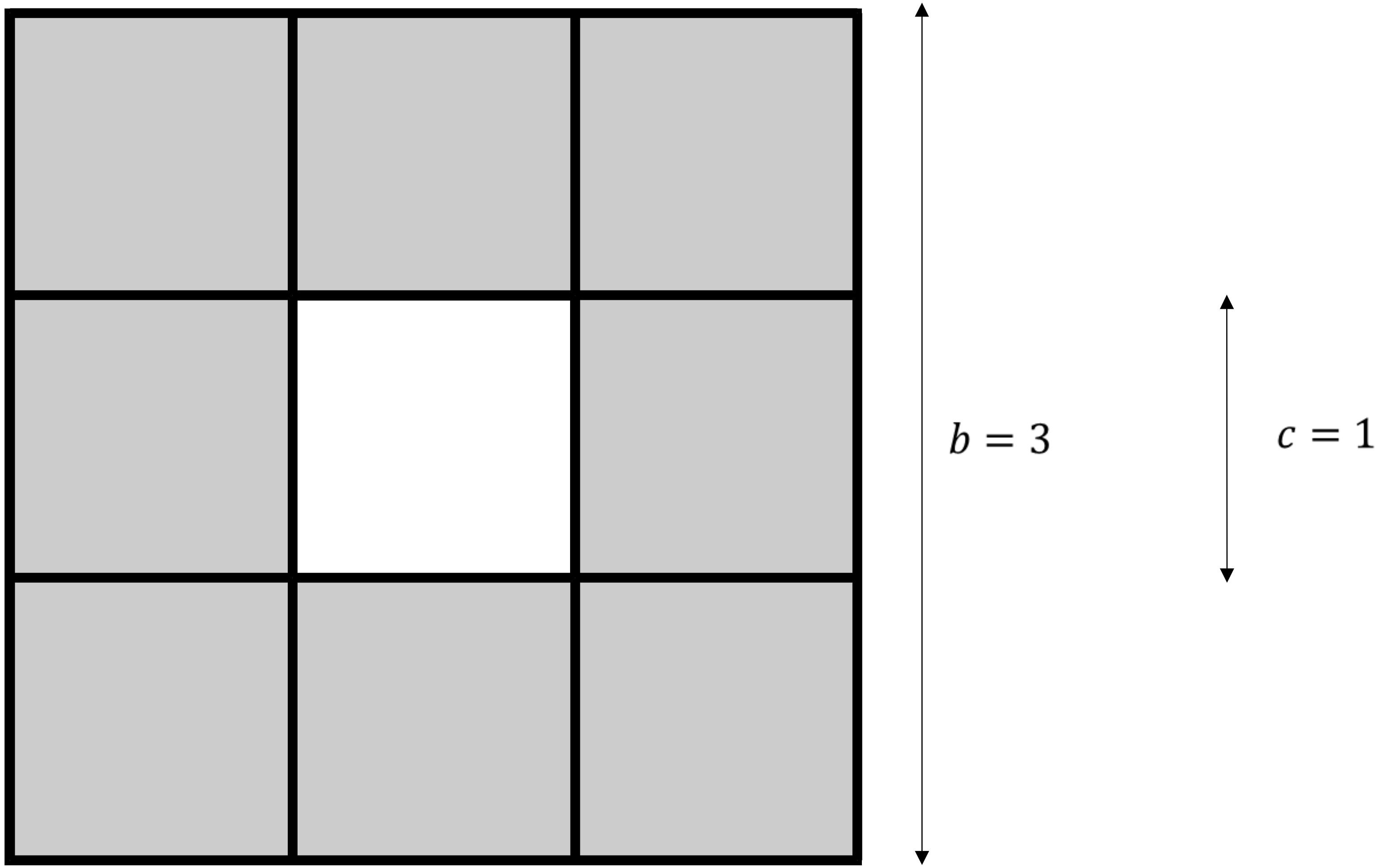
Sierpinski carpet

$l = 0$



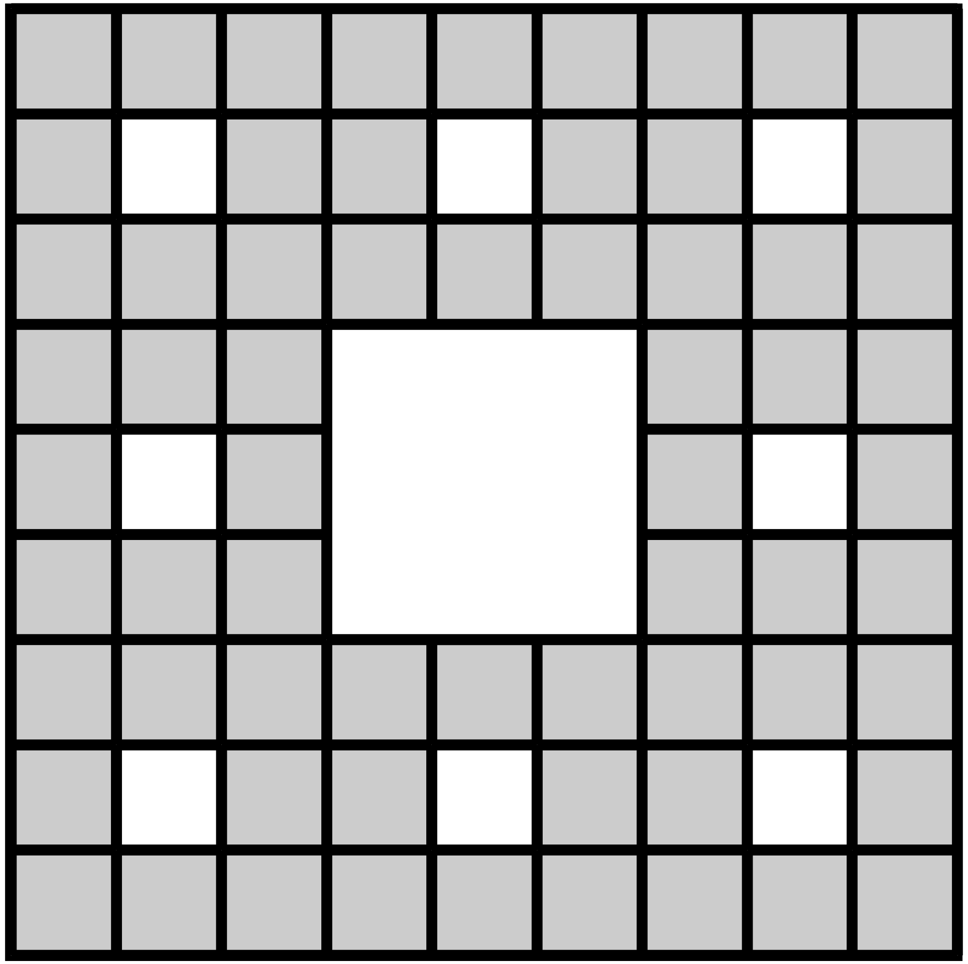
Sierpinski carpet

$l = 1$



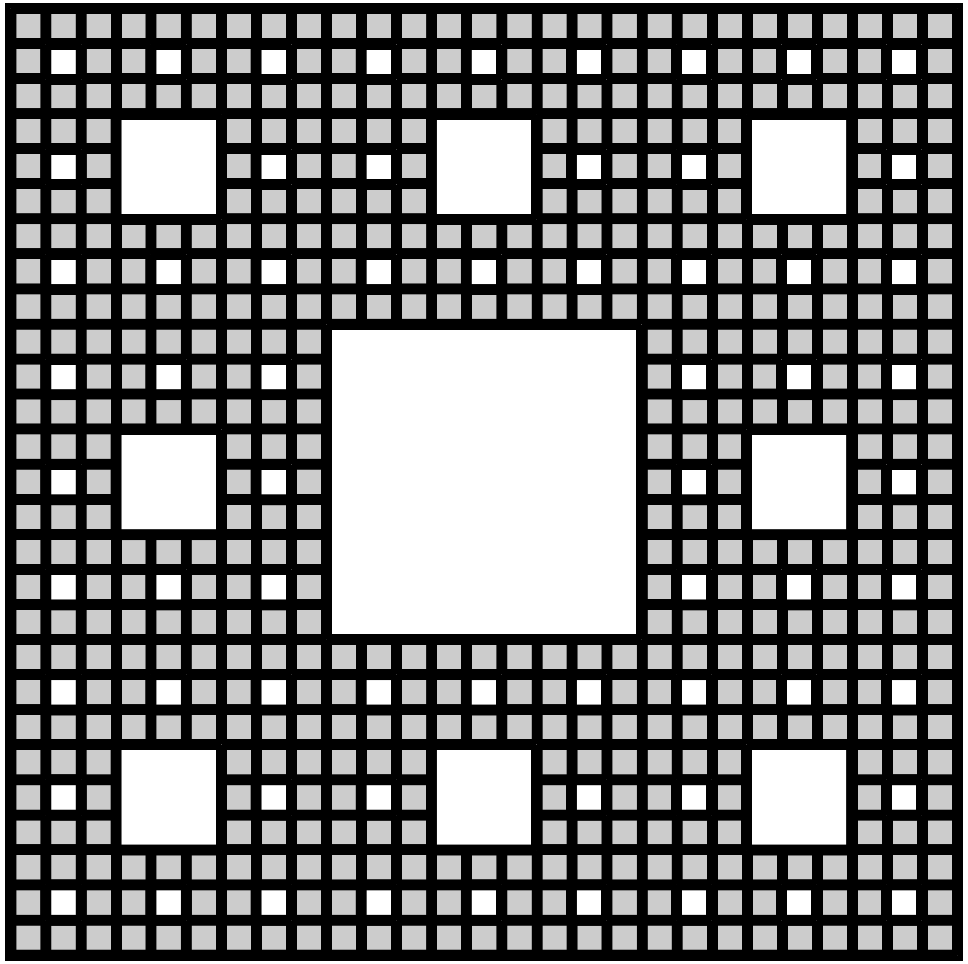
Sierpinski carpet

$l = 2$



Sierpinski carpet

$l = 3$



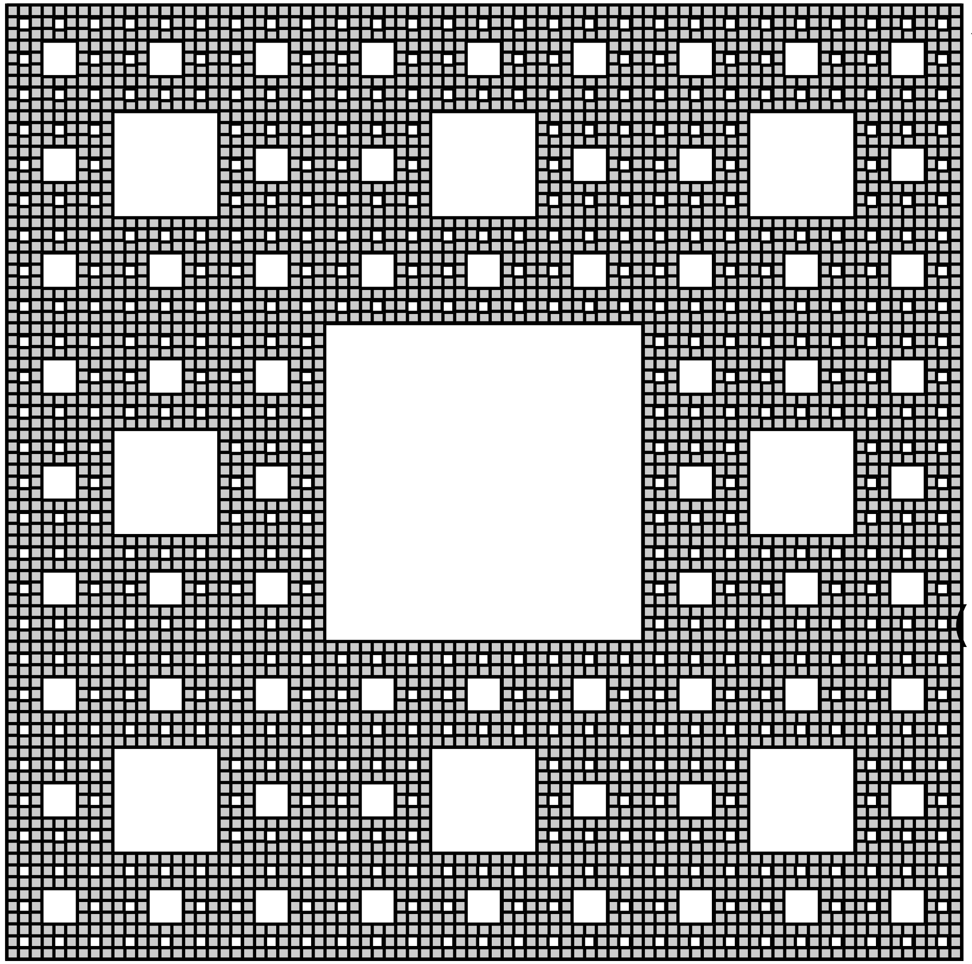
Sierpinski carpet

Sierpinski carpet graph

$$l = 4$$

$$d = \frac{\ln(b^2 - c^2)}{\ln(b)}$$

$$1 < d < 2$$



Thermally stable Ising model

(Vezzani, cond-mat/0212497)

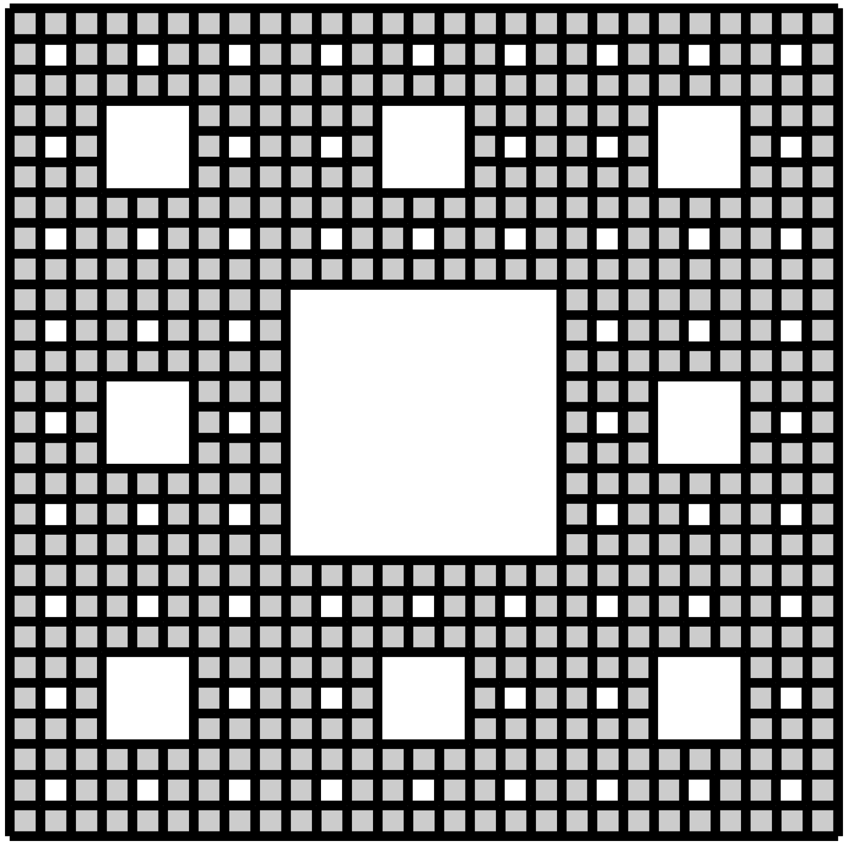
(Shinoda, J. Appl. Prob., 39, 1, 200

Fractal Product Codes

(Freedman, Hastings 1301.1363)

Homological product

(Bravyi, Hastings 1311.0885)



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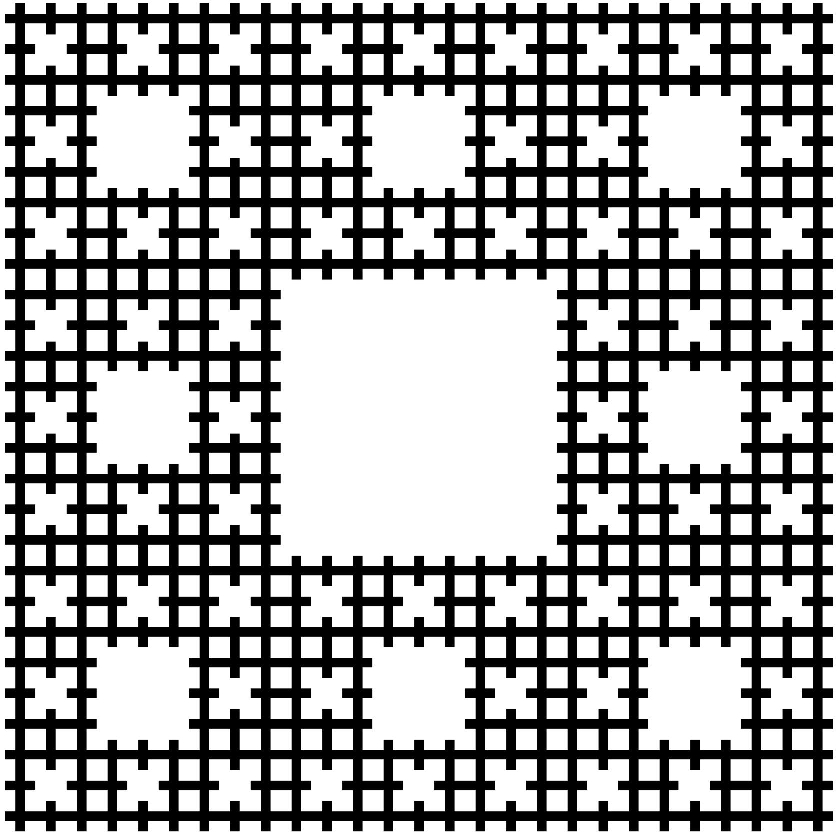

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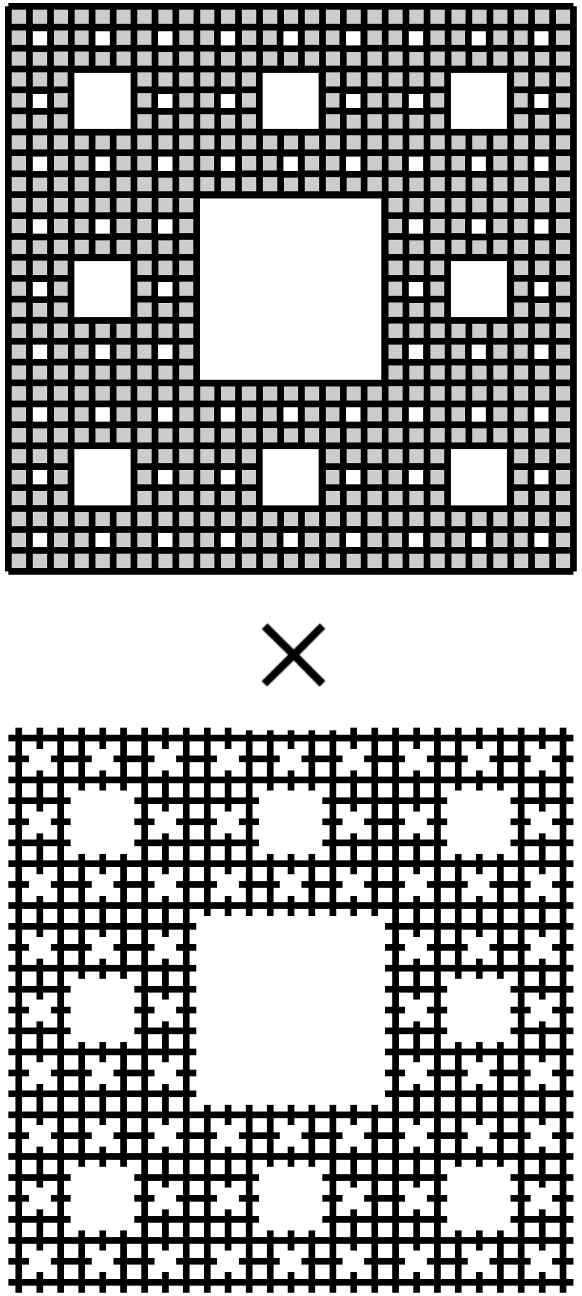
4D complex – subgraph of hypercubic lattice

- Qubits on faces
- X-type stabilizers on links
- Z-type stabilizers on cubes

FPC = 4D toric code with punctures

$$H = -\sum_{l} A_l - \sum_{c} B_c$$

Hausdorff dimension < 3



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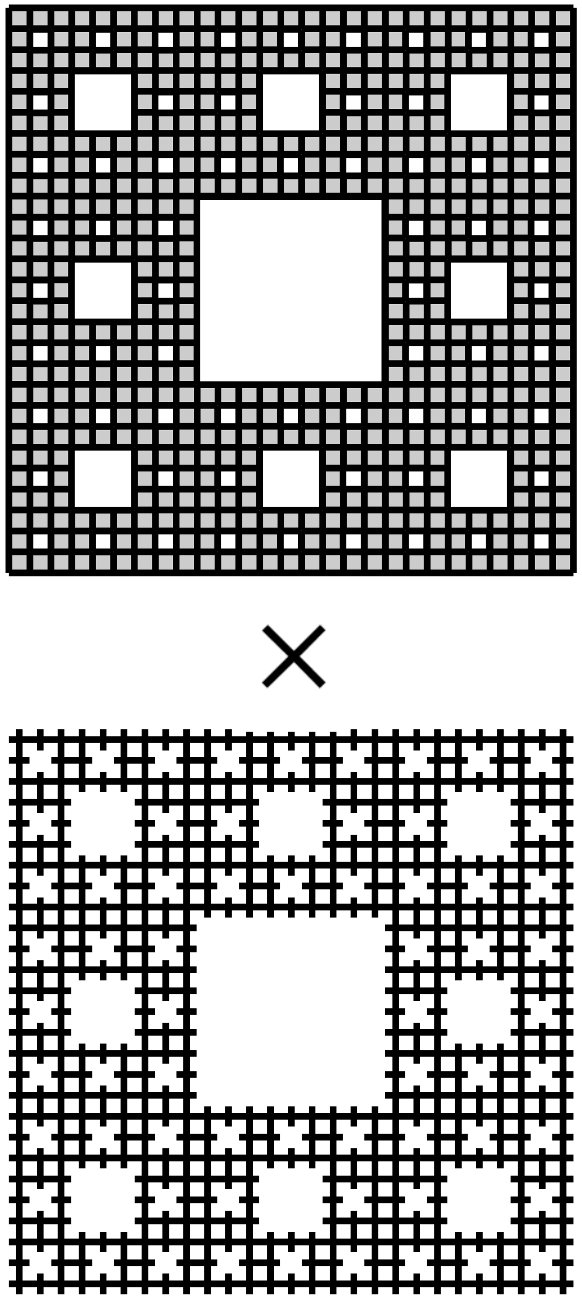
Degeneracy (from Künneth formula)

- 1 global encoded qubit
- $\left(\frac{1-\left(b^2-c^2\right)^l}{1-\left(b^2-c^2\right)}\right)^2$ local encoded qubits



Thermodynamic properties of FPCs

- $$H_{FPC} = -\sum_l A_l - \sum_c B_c$$

$$H_{FPC}^X = -\sum_l A_l \qquad\qquad H_{FPC}^Z = -\sum_c B_c$$

The two classical models are related by a rotation symmetry.

Main result: H_{FPC} has (at least) 2 phase transitions

Correlation inequalities

- Generalized ferromagnetic Ising models

$$H = -\sum_R J_R B_R$$

GKS inequality: (Griffiths, J. Math. Phys. 8, 478, 1967) (Kelly, Sherman, J. Math. Phys. 9, 466, 1968)

$$\frac{\partial}{\partial J_R} \langle B_{R'} \rangle = \langle B_R B_{R'} \rangle - \langle B_R \rangle \langle B_{R'} \rangle \geq 0$$

You can't destroy ferromagnetic correlations with ferromagnetic terms

Duality transformations

- Merlini-Gruber duality: (Merlini, Gruber, J. Math. Phys. 13, 1814, 1972)

  Construct a system Λ^* dual to Λ, such that
  $$\mathcal{Z}(\Lambda^*) \propto \mathcal{Z}(\Lambda)$$
  phase transition in $\Lambda \Leftrightarrow$ phase transition in Λ^*

  Qubits of Λ^* = constraints of Λ

  Interactions of Λ^* between constraints of Λ that share a stabilizer

Idea of thermodynamic analysis

- Merlini-Gruber duality transformations
  - Finds related models with equivalent phase structure
- GKS correlation inequality
  - Adding ferromagnetic terms can't destroy ferromagnetic correlations

- The dual model to $H^{Z}_{FPC} = -\sum_c B_c$ is an Ising model on SC×SC (in bulk)
- A submodel of it is an Ising model on SC
- This has a finite temperature phase transition

Conclusion: FPC has two phase transitions

Evidence of self-correction?

Fractal product codes - Caltech checklist

1. Finite spins
2. Bounded local interactions
3. Nontrivial codespace
4. Perturbative stability
5. Efficient decoding
6. Exponential lifetime

} In $\mathbb{R}^3$ - random projections?

Künneth formula

~ topological stability theorems

~ 4D toric code decoders

~ phase transitions

Than